Parallel Matlab programming using Distributed Arrays

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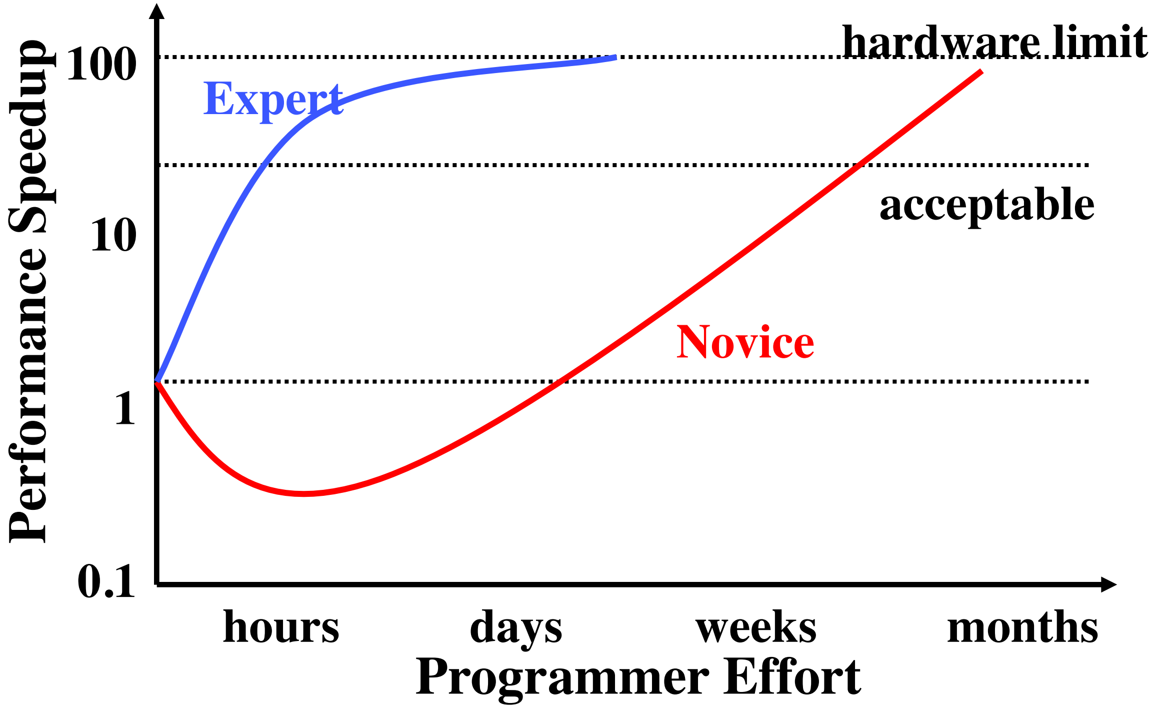
Goal: Think Matrices not Messages

- In the past, writing well performing parallel programs has required a lot of code and a lot of expertise
- pMatlab distributed arrays eliminates the coding burden
  - However, making programs run fast still requires expertise
- This talk illustrates the key math concepts experts use to make parallel programs perform well



Outline

- **Parallel Design**
- **Distributed Arrays**
- **Concurrency vs Locality**
- **Execution**
- **Summary**

- ***Serial Program***
- ***Parallel Execution***
- ***Distributed Arrays***
- ***Explicitly Local***

Serial Program

Math	Matlab
$\mathbf{X},\mathbf{Y} : \mathbb{R}^{N\mathrm{x}N}$	`X = zeros(N,N);` `Y = zeros(N,N);`
$\mathbf{Y} = \mathbf{X} + 1$	`Y(:,:) = X + 1;`

- **Matlab is a high level language**
- **Allows mathematical expressions to be written concisely**
- **Multi-dimensional arrays are *fundamental* to Matlab**

Parallel Execution

Math

$P_{ID}=N_P-1$

$P_{ID}=1$

$P_{ID}=0$

$$\mathbf{X},\mathbf{Y} : \mathbb{R}^{NxN}$$

$$\mathbf{Y} = \mathbf{X} + 1$$

pMatlab

Pid=Np-1

Pid=1

Pid=0

```
X = zeros(N,N);
Y = zeros(N,N);

Y(:,:) = X + 1;
```

- **Run N_P (or `Np`) copies of same program**
  - **Single Program Multiple Data (SPMD)**
- **Each copy has a unique P_{ID} (or `Pid`)**
- **Every array is *replicated* on each copy of the program**

Distributed Array Program

Math

$P_{ID}=N_P-1$

$P_{ID}=1$

$P_{ID}=0$

$$\mathbf{X},\mathbf{Y} : \mathbb{R}^{P(N)xN}$$

$$\mathbf{Y} = \mathbf{X} + 1$$

pMatlab

Pid=Np-1

Pid=1

Pid=0

```
XYmap = map([Np N1],{},0:Np-1);
X = zeros(N,N,XYmap);
Y = zeros(N,N,XYap);

Y(:,:) = X + 1;
```

- **Use P() notation (or `map`) to make a distributed array**
- **Tells program which dimension to distribute data**
- **Each program implicitly operates on only its own data (owner computes rule)**

Explicitly Local Program

Math	pMatlab

$\mathbf{X},\mathbf{Y} : \mathbb{R}^{P(N)xN}$	`XYmap = map([Np 1],{},0:Np-1);`
	`Xloc = local(zeros(N,N,XYmap));`
	`Yloc = local(zeros(N,N,XYmap));`

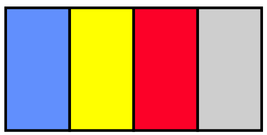
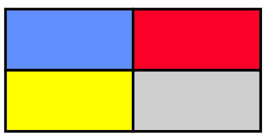
$\mathbf{Y}.\mathrm{loc} = \mathbf{X}.\mathrm{loc} + 1$	`Yloc(:,:) = Xloc + 1;`

- **Use** $.\mathrm{loc}$ **notation (or** `local` **function) to explicitly retrieve local part of a distributed array**
- **Operation is the same as serial program, but with different data on each processor (recommended approach)**

Outline

- Parallel Design
- **Distributed Arrays**
  - *Maps*
  - *Redistribution*
- Concurrency vs Locality
- Execution
- Summary

Parallel Data Maps

Array	Math	Matlab
	$\mathbb{R}^{P(N)xN}$	`Xmap=map([Np 1],{},0:Np-1)`
	$\mathbb{R}^{NxP(N)}$	`Xmap=map([1 Np],{},0:Np-1)`
	$\mathbb{R}^{P(N)xP(N)}$	`Xmap=map([Np/2 2],{},0:Np-1)`

Computer

P_{ID}	0	1	2	3	Pid

- **A map is a mapping of array indices to processors**
- **Can be block, cyclic, block-cyclic, or block w/overlap**
- **Use P() notation (or `map`) to set which dimension to split among processors**

Maps and Distributed Arrays

A processor *map* for a numerical array is an *assignment of blocks of data to processing elements*.

```
Amap = map([Np 1],{},0:Np-1);
```

Processor Grid

Distribution
{}=default=block

List of processors

```
A = zeros(4,6,Amap);
```

P0
P1
P2
P3

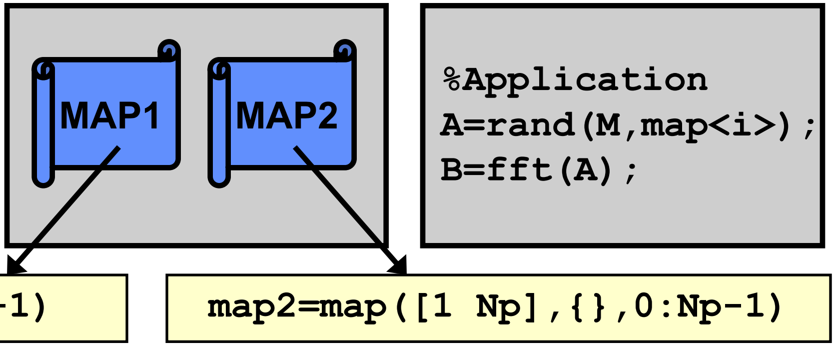
pMatlab constructors are overloaded to take a `map` as an argument, and return a distributed array.

A =

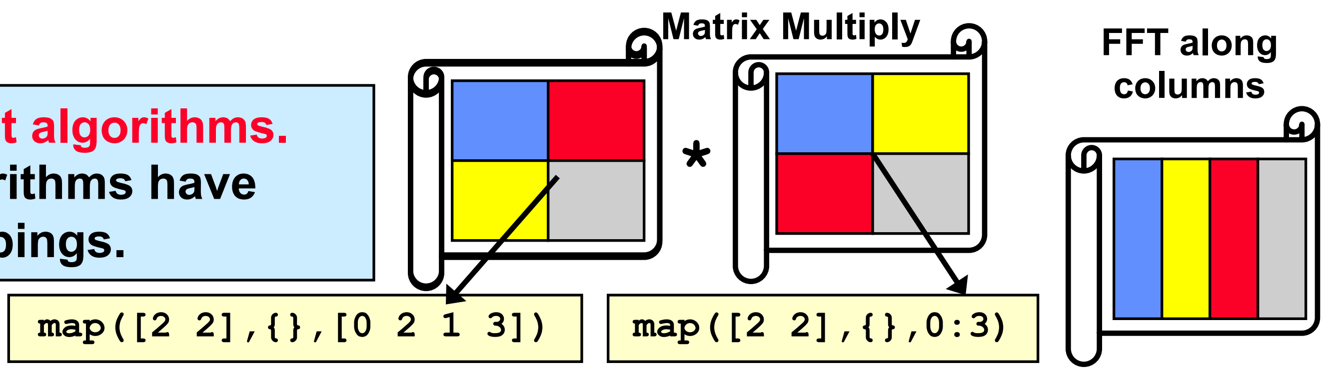
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Advantages of Maps

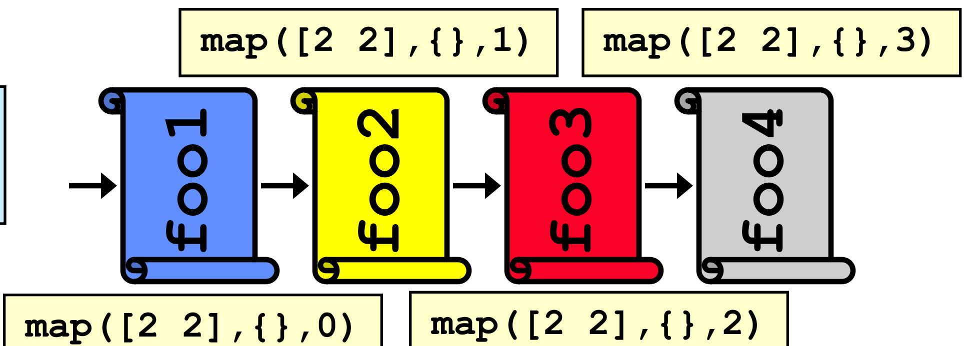
Maps are scalable. Changing the number of processors or distribution does not change the application.



Maps support different algorithms. Different parallel algorithms have different optimal mappings.



Maps allow users to set up pipelines in the code (implicit task parallelism).



Redistribution of Data

Math

$\mathbf{X} \quad : \mathbb{R}^{P(N)xN}$

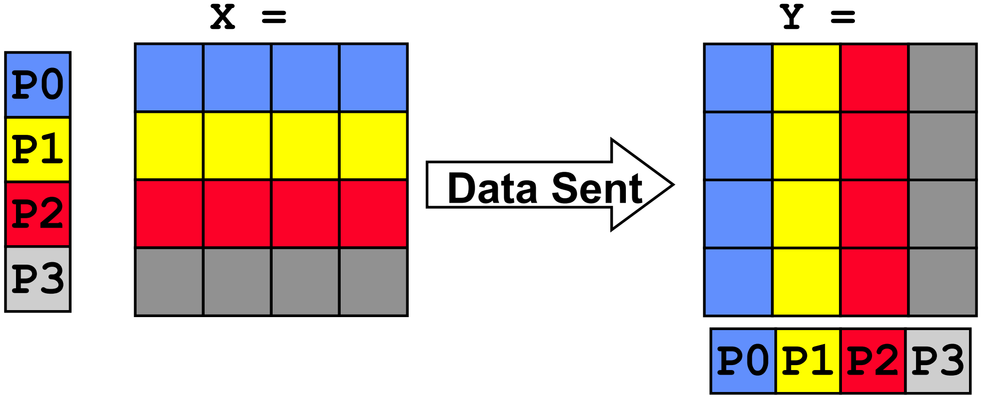
$\mathbf{Y} \quad : \mathbb{R}^{NxP(N)}$

$$\mathbf{Y} = \mathbf{X} + 1$$

pMatlab

```
Xmap = map([Np 1],{},0:Np-1);
Ymap = map([1 Np],{},0:Np-1);
X = zeros(N,N,Xmap);
Y = zeros(N,N,Ymap);

Y(:,:) = X + 1;
```



- Different distributed arrays can have different maps
- Assignment between arrays with the "=" operator causes data to be redistributed
- Underlying library determines all the message to send

Outline

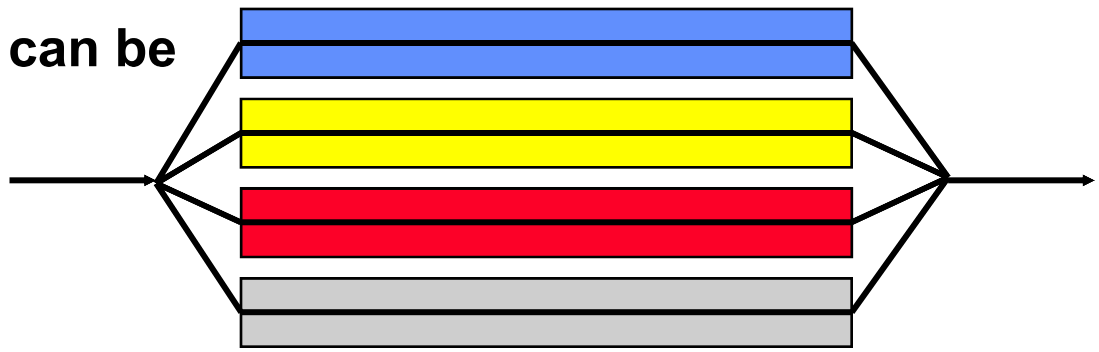
- Parallel Design
- Distributed Arrays
- **Concurrency vs Locality**
  - ***Definition***
  - ***Example***
  - ***Metrics***
- Execution
- Summary

Definitions

Parallel Concurrency

- **Number of operations that can be done in parallel (i.e. no dependencies)**
- **Measured with:**

  Degrees of Parallelism

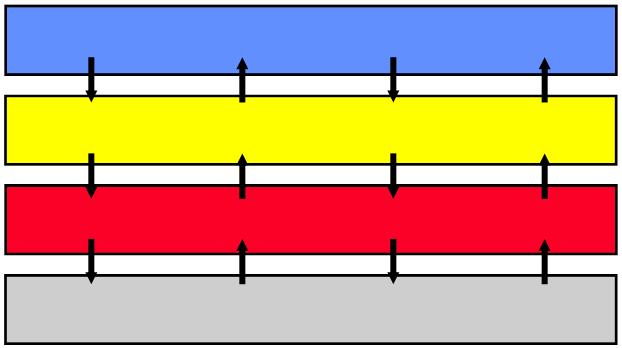


Parallel Locality

- **Is the data for the operations local to the processor**
- **Measured with ratio:**

  Computation/Communication

  = (Work)/(Data Moved)



- **Concurrency is ubiquitous; "easy" to find**
- **Locality is harder to find, but is the key to performance**
- ***Distributed arrays derive concurrency from locality***

Serial

Math

$\mathbf{X},\mathbf{Y} : \mathbb{R}^{NxN}$

for i=1:N
  for j=1:N
    $\mathbf{Y}(i,j) = \mathbf{X}(i,j) + 1$

Matlab

```
X = zeros(N,N);
Y = zeros(N,N);

for i=1:N
  for j=1:N
    Y(i,j) = X(i,j) + 1;
  end
end
```

- **Concurrency: max degrees of parallelism = N^2**
- **Locality**
  - **Work = N^2**
  - **Data Moved: depends upon map**

1D distribution

Math	pMatlab

Math

$\mathbf{X},\mathbf{Y} : \mathbb{R}^{P(N)xN}$

for i=1:N
  for j=1:N
    $\mathbf{Y}(i,j) = \mathbf{X}(i,j) + 1$

pMatlab

```
XYmap = map([NP 1],{},0:Np-1);
X = zeros(N,N,XYmap);
Y = zeros(N,N,XYmap);

for i=1:N
  for j=1:N
    Y(i,j) = X(i,j) + 1;
  end
end
```

- **Concurrency: degrees of parallelism = min(N,N_P)**
- **Locality: Work = N^2, Data Moved = 0**
- **Computation/Communication = Work/(Data Moved) $\rightarrow \infty$**

2D distribution

Math

$\mathbf{X},\mathbf{Y} : \mathbb{R}^{P(N) x P(N)}$

for i=1:N
  for j=1:N
    $\mathbf{Y}(i,j) = \mathbf{X}(i,j) + 1$

pMatlab

```
XYmap = map([Np/2 2],{},0:Np-1);
X = zeros(N,N,XYmap);
Y = zeros(N,N,XYmap);

for i=1:N
  for j=1:N
    Y(i,j) = X(i,j) + 1;
  end
end
```

- **Concurrency: degrees of parallelism = min(N^2,N_P)**
- **Locality: Work = N^2, Data Moved = 0**
- **Computation/Communication = Work/(Data Moved) $\rightarrow \infty$**

2D Explicitly Local

Math

$\mathbf{X},\mathbf{Y} : \mathbb{R}^{P(N)xP(N)}$

for i=1:size(**X**.loc,1)
  for j=1:size(**X**.loc,2)
    Y.loc(i,j) =
      X.loc(i,j) + 1

pMatlab

```
XYmap = map([Np/2 2],{},0:Np-1);
Xloc = local(zeros(N,N,XYmap));
Yloc = local(zeros(N,N,XYmap));

for i=1:size(Xloc,1)
  for j=1:size(Xloc,2)
    Yloc(i,j) = Xloc(i,j) + 1;
  end
end
```

- **Concurrency: degrees of parallelism = min(N^2,N_P)**
- **Locality: Work = N^2, Data Moved = 0**
- **Computation/Communication = Work/(Data Moved) $\rightarrow \infty$**

1D with Redistribution

Math

$\mathbf{X} : \mathbb{R}^{P(N)xN}$

$\mathbf{Y} : \mathbb{R}^{NxP(N)}$

for i=1:N
  for j=1:N
    Y(i,j) = **X**(i,j) + 1

pMatlab

```
Xmap = map([Np 1],{},0:Np-1);
Ymap = map([1 Np],{},0:Np-1);
X = zeros(N,N,Xmap);
Y = zeros(N,N,Ymap);


for i=1:N
  for j=1:N
    Y(i,j) = X(i,j) + 1;
  end
end
```

- **Concurrency: degrees of parallelism = min(N,N_P)**
- **Locality: Work = N^2, Data Moved = N^2**
- **Computation/Communication = Work/(Data Moved) = 1**

Outline

- Parallel Design
- Distributed Arrays
- Concurrency vs Locality
- **Execution**
  - *Four Step Process*
  - *Speedup*
  - *Amdahl's Law*
  - *Performfmance vs Effort*
  - *Portability*
- Summary

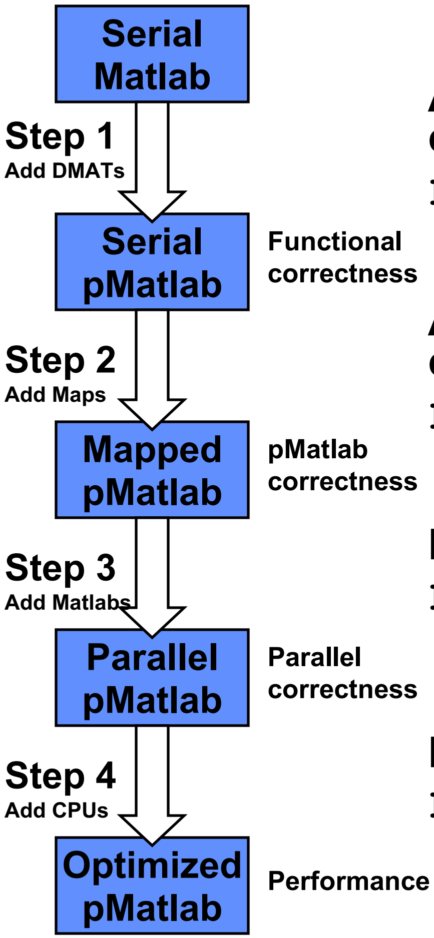
Running

- **Start Matlab**
  - Type: `cd examples/AddOne`
- **Run dAddOne**
  - Edit `pAddOne.m` and set: `PARALLEL = 0;`
  - Type: `pRUN('pAddOne',1,{})`
- **Repeat with:** `PARALLEL = 1;`
- **Repeat with:** `pRUN('pAddOne',2,{});`
- **Repeat with:** `pRUN('pAddOne',2,{'cluster'});`

- **Four steps to taking a serial Matlab program and making it a parallel Matlab program**

Parallel Debugging Processes

- Simple four step process for debugging a parallel program



Add distributed matrices without maps, verify functional correctness

```
PARALLEL=0;    pRUN('pAddOne',1,{});
```

Add maps, run on 1 processor, verify parallel correctness, compare performance with Step 1

```
PARALLEL=1;    pRUN('pAddOne',1,{});
```

Run with more processes, verify parallel correctness

```
PARALLEL=1;    pRUN('pAddOne',2,{}) );
```

Run with more processors, compare performance with Step 2

```
PARALLEL=1;    pRUN('pAddOne',2,{'cluster'});
```

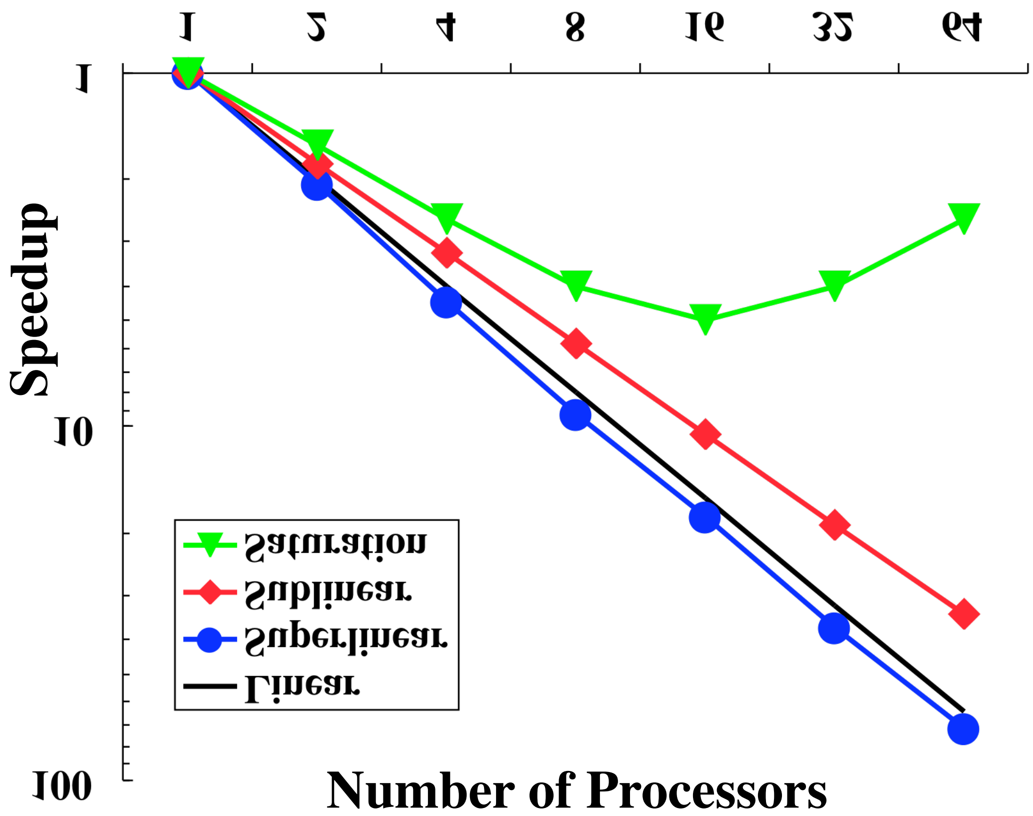
- **Always debug at earliest step possible (takes less time)**

Timing

- **Run dAddOne:** `pRUN('pAddOne',1,{'cluster'});`
  - Record `processing_time`
- **Repeat with:** `pRUN('pAddOne',2,{'cluster'});`
  - Record `processing_time`
- **Repeat with:** `pRUN('pAddone',4,{'cluster'});`
  - Record `processing_time`
- **Repeat with:** `pRUN('pAddone',8,{'cluster'});`
  - Record `processing_time`
- **Repeat with:** `pRUN('pAddone',16,{'cluster'});`
  - Record `processing_time`

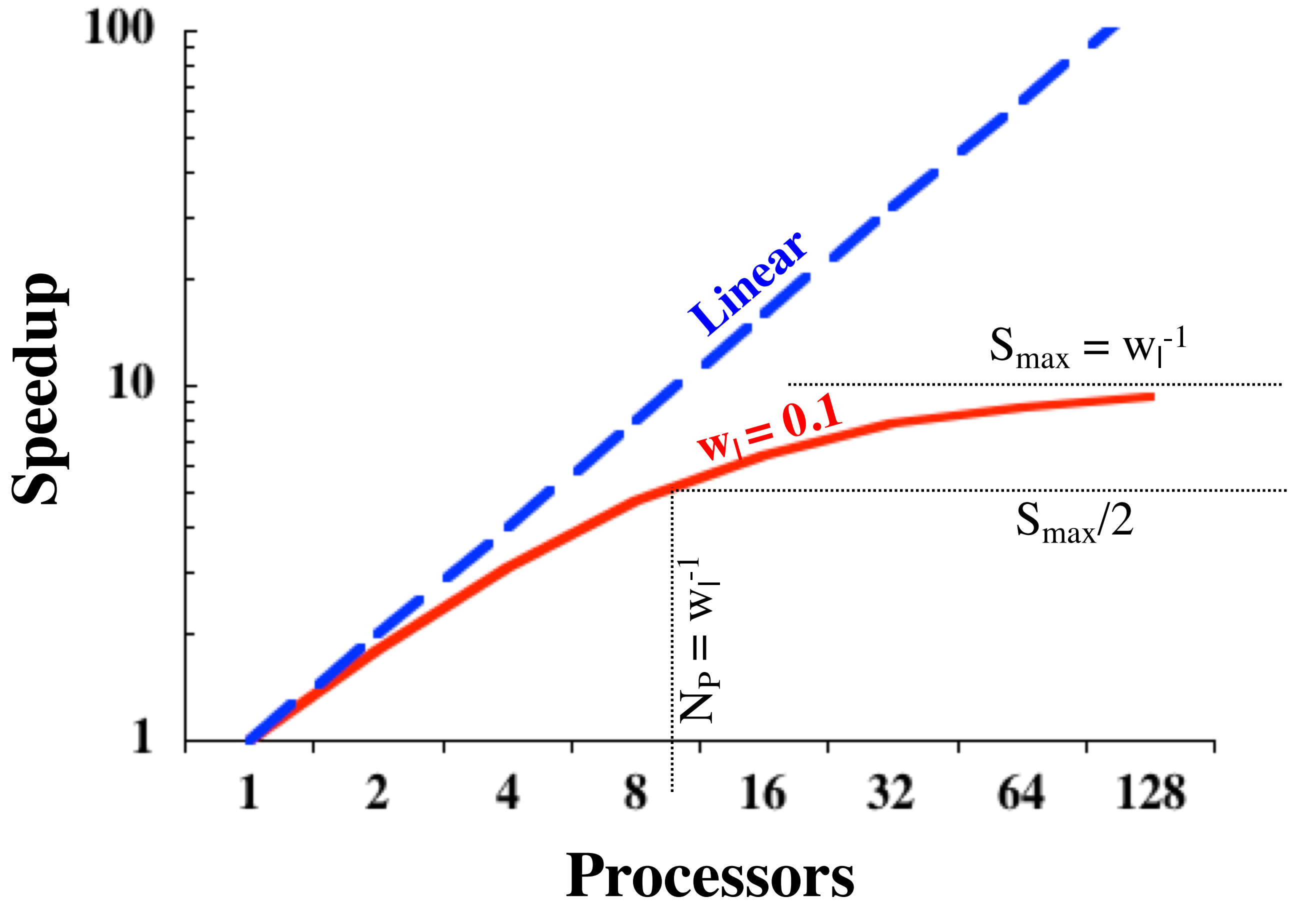
- **Run program while doubling number of processors**
- **Record execution time**

Computing Speedup



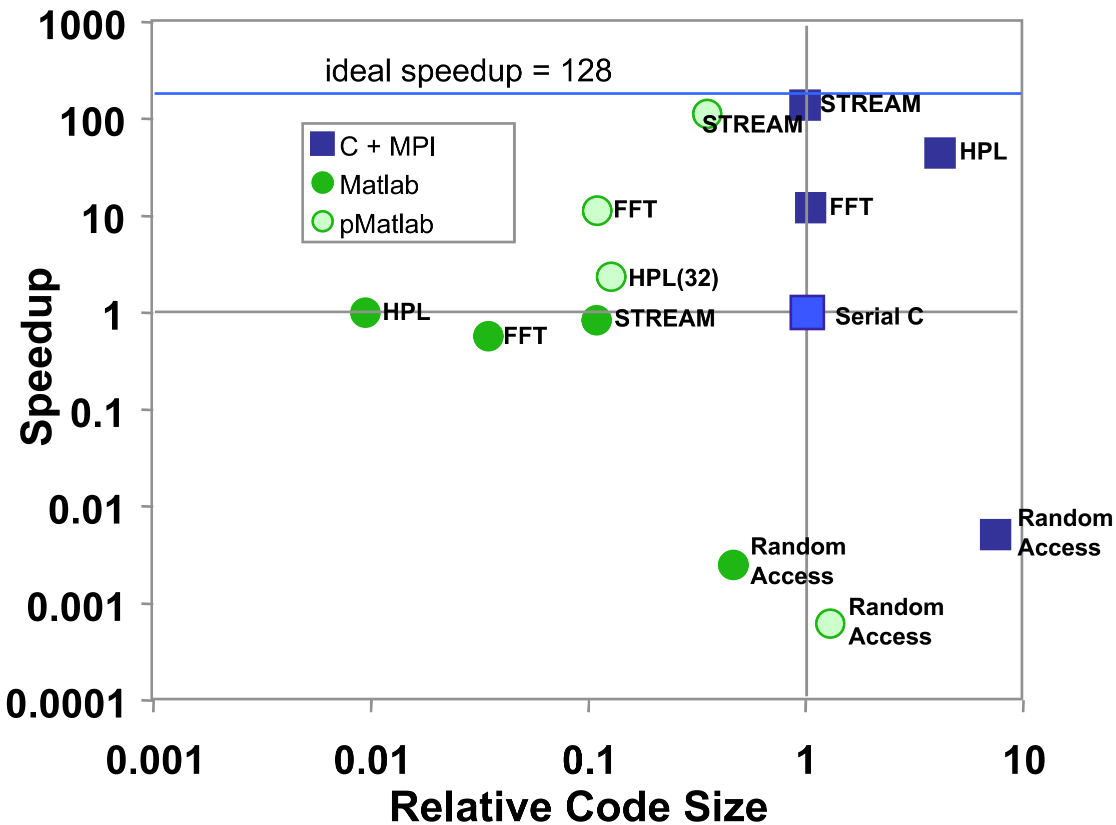
- Speedup Formula: Speedup(N_P) = Time(N_P=1)/Time(N_P)
- Goal is sublinear speedup
- All programs saturate at some value of N_P

Amdahl's Law



- Divide work into parallel ($w_{||}$) and serial ($w_|$) fractions
- Serial fraction sets maximum speedup: $S_{max} = w_|^{-1}$
- Likewise: Speedup($N_P = w_|^{-1}$) = $S_{max}/2$

HPC Challenge Speedup vs Effort



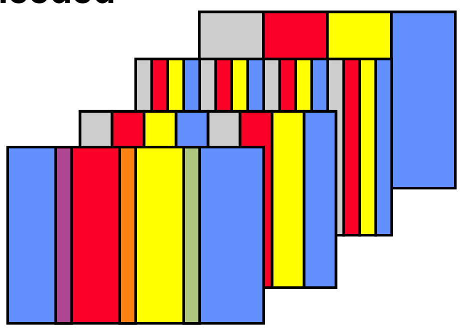
- Ultimate Goal is speedup with minimum effort
- HPC Challenge benchmark data shows that pMatlab can deliver high performance with a low code size

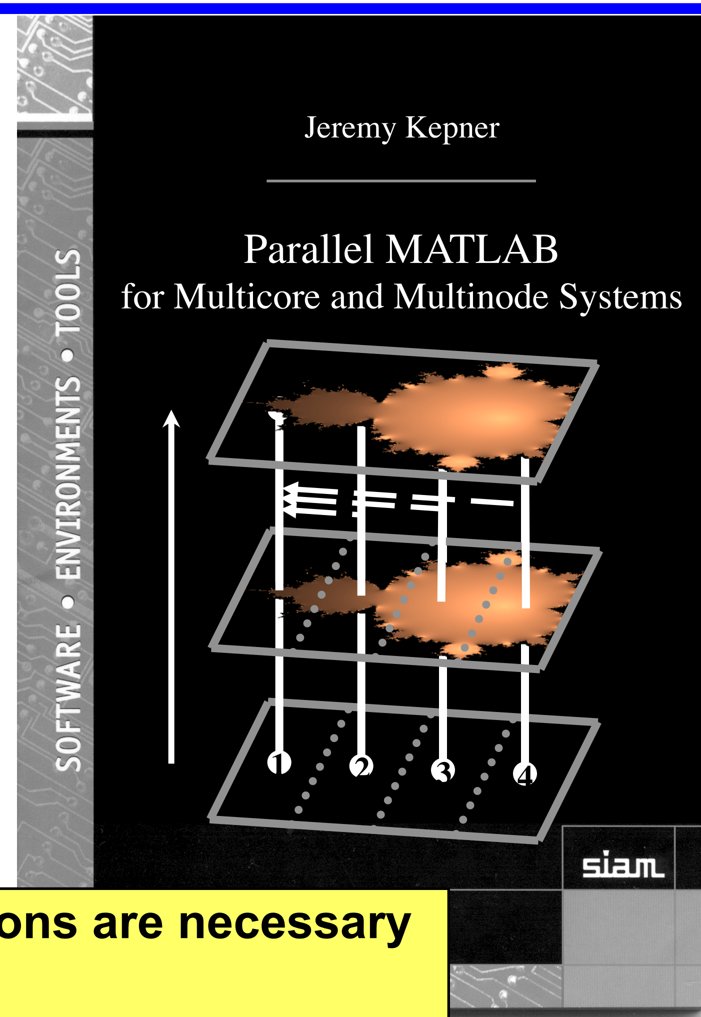
Portable Parallel Programming

Universal Parallel Matlab programming

```
Amap = map([Np 1],{},0:Np-1);
Bmap = map([1 Np],{},0:Np-1);
A = rand(M,N,Amap);
B = zeros(M,N,Bmap);
B(:,:) = fft(A);
```

- pMatlab runs in all parallel Matlab environments
- Only a few functions are needed
  - Np
  - Pid
  - map
  - local
  - put_local
  - global_index
  - agg
  - SendMsg/RecvMsg





- **Only a small number of distributed array functions are necessary to write nearly all parallel programs**
- **Restricting programs to a small set of functions allows parallel programs to run efficiently on the widest range of platforms**

Summary

- Distributed arrays eliminate most parallel coding burden
- Writing well performing programs requires expertise
- Experts rely on several key concepts
  - Concurrency vs Locality
  - Measuring Speedup
  - Amdahl's Law
- Four step process for developing programs
  - Minimizes debugging time
  - Maximizes performance