

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
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AUTOMATIC REPORTING OF HEIGHT (AROH)  
DESIGN AND TRADE-OFF STUDIES

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### ABSTRACT

Application of MTD signal processing and state-of-the-art data processing can result in a completely automatic nodding beam height finder. The resulting savings in manpower are significant. Calculations show that such a system should have good sensitivity and adequate rejection of ground and weather clutter. Modification of an FPS-6 radar for this purpose is discussed.

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## I. INTRODUCTION AND SUMMARY

The FPS-6 height finder was designed in the early 1950's. It is a straightforward, simple, reliable radar which has been in use, pretty much unchanged, continuously right down to the present day. Being a height finder, the FPS-6 is normally used in conjunction with a search radar. The latter locates aircraft in the dimensions of range and azimuth and the height finder is used to provide altitude information on selected targets. Normally one or more height finders are located adjacent to the associated search radar in order to avoid parallax errors.

The FPS-6 was designed to operate with its transmitter slaved to that of the search radar. The master trigger timing for the search radar was derived from the delay line in the search MTI system. This same trigger was used to synchronize the search radar transmitter, the height finder radar transmitter and all analog displays at the site. This was done in part to reduce the effects of interference between the different radar sets, but primarily so that radar range information could be handed back and forth between different radars in the form of analog delays which were all referenced to the same master timing trigger.

The elevation drive in the FPS-6 is a straightforward and basically reliable mechanism. A relatively large induction motor turns a crank through a gear train. The crank is linked to the antenna through a connecting rod so that the cranking action causes the antenna to nod up and down. This nodding goes on continually over the entire elevation coverage.

The height radar operator is presented with an intensity modulated cathode ray tube display (the height range indicator - HRI). The sweep of the display is arranged so that the horizontal position of a target return indicates the target range and the vertical position indicates the height of the target above the ground. The sweeps are necessarily not linear. The display acts essentially as an analog computer. It accepts range and elevation angle information from the radar and converts it into range and height information for the operator. The sweep waveforms are adjusted to compensate for the effects of the earth's curvature and of atmospheric refraction.

The FPS-6 is operated as a non-coherent radar. It provides only normal video information to the operator's range height indicator. It is a very high

powered system however and can produce ample signal-to-noise ratios even at the longer ranges. On the HRI the radar return information from a single aircraft target is presented in the form of 8 to 25 or so relatively bright, single pulse spots. The signal-to-noise ratio is high enough so that these single pulse returns are readily visible in areas where no precipitation or ground returns are cluttering up the display.

All the echoes from a single target are spread out vertically over a distance on the HRI face which may range from less than 1/4 inch to more than 3/4 of an inch. The operator has been trained to detect these target configurations and to estimate the vertical centroid. With nominal training and minimal experience an operator can resolve targets from thermal noise quite successfully. The probability of detection, hence of successful height determination, falls off rapidly in areas cluttered by returns from the earth's surface or from precipitation. Some more experienced operators can pick targets out of light precipitation or ground clutter, but the probability of detection and the accuracy of the results are seriously degraded in heavier clutter.

In normal operation the height finder antenna is slewed in azimuth to the direction of the target. Then the operator is given a pair of electronic cursors on the RHI display. A vertical cursor is positioned horizontally to indicate the range of the target and a horizontal one which is positioned vertically to indicate whatever a priori height information is available. If no target is seen the operator makes a search over nearby azimuths. Upon detecting the target, the operator moves the cursors so that they intersect at the centroid of the target returns. He then informs the users that height data is ready. In the SAGE and JSS systems the user is a computer center which is located many miles from the radar site. The height requests and radar heights are transmitted to and from the remote computer center via voice circuits and modems. A portion of the FYQ-47 (so called "production common digitizer") is devoted to receiving height requests from the direction centers, converting them into appropriate analog signals and transmitting them to the height finder as well as formatting the height reports and transmitting them back to the direction center.

Thus, the FPS-6 height finder combined with the operator and the FYQ-47 can be regarded as a sensor subsystem in which all filtering, thresholding and parameter estimation functions are performed by the operator.

In recent years the cost of maintaining a cadre of operators has become prohibitive. Meanwhile, sophisticated, reliable, digital hardware for signal and data processing has become relatively plentiful and inexpensive. Lincoln Laboratory has demonstrated a practical digital signal processor on an E-band search radar. This processor, known as the Moving Target Detector (MTD), provides automatic detection with excellent detection statistics in all weather conditions encountered. It is expected that the application of MTD principles to a height finding radar will result in a completely automatic height reporting system. The objective of the AROH project is the breadboarding and feasibility demonstration of just such a system.

The design of this system can be partitioned into three interrelated areas; namely, (1) the design of the radar signal waveform and scanning strategy, (2) the design of the signal processor to provide optimum or near-optimum detection and parameter estimation on the received signals and (3) the design of a radar transmitter, receiver and antenna to generate, transmit and receive the signal. In the AROH project, since the use of an FPS-6 radar has been decided upon, the third design area becomes the designing of modifications to the existing radar so that it will be compatible with the rest of the system. Also in AROH the signal design may be constrained by radar hardware limitations.

The FPS-6 is a reliable and powerful but primitive radar. No signal processing whatsoever takes place in the current FPS-6. Except for the IF bandwidth, all filtering, detection thresholding, clutter rejection and parameter estimation in the SAGE height finding system takes place in the eyes and brain of the height finder operator. The digital processing portions of the AROH will be made up of state-of-the-art hardware and software. However, they will obviously be quite a bit less sophisticated than the brain of even a mediocre operator. Thus, if the automatic system is to perform as well as the manually operated FPS-6, considerable refinement will have to take place at the radar before the addition of the digital signal processor. The overall result of these refinements will be

to increase the resolving power of the basic radar; i.e., to make the basic resolution cell of the radar smaller.

Non-coherent radar, when properly designed, can resolve individual radar echoes from each other in three dimensions or coordinates. These coordinates are azimuth, elevation and range. Coherent radars, either fully coherent or coherent-on-receive, can resolve in the fourth dimension of radial velocity. Radial velocity appears at the radar as a doppler offset or a difference between the frequency of the received echo and that of the transmitted pulse. Thus, the non-coherent FPS-6 can resolve two targets at the same range if they are sufficiently far apart in either azimuth or elevation. The addition of a coherent receiving channel makes it possible for the radar to resolve two returns at the same range, azimuth and elevation if they differ sufficiently in radial velocity. The size of the resolution cell in the dimensions of azimuth and elevation is determined by the radar antenna pattern. The extent in range of the resolution cell is limited by the radar pulse width and the size of the radial velocity resolution cell is limited by the radar signal frequency and the time on target. The use of a uniform train of pulses as is done in the FPS-6 and the AROH system gives rise to ambiguities in range and in radial velocity (see references 1 and 2).

In modifying the FPS-6 for use in the AROH system the pulse width will be reduced from 2 to 1 microsecond. A coherent-on-receive IF channel will be added to the receiver so that resolution in the doppler (i.e., radial velocity) dimension will be possible. The time-on-target will be 32 radar pulses so that the doppler processor can produce 32 doppler channels. (The period necessary to collect the information from the 32 pulses is known as a coherent processing interval - CPI.) The combination of reduced pulse width and doppler filtering into 32 channels has the effect of dividing the original range azimuth elevation cell into no less than 64 range azimuth elevation doppler cells. Thus, where the operator was presented a single output on the HRI face the AROH data processor will be given 64 independent outputs. This large number of independent channels is better suited to automatic processing. Hence, we expect to design a practical digital machine which can be expected to cope with the output of the modified FPS-6 as well or better than a standard operator could cope with the output from the unmodified radar.

We have more or less arbitrarily divided the digital processing functions into the areas of signal processing and data processing. Signal processing, for the purposes of the AROH project, denotes all the processing functions from the analog-to-digital converters through the thresholds. This includes clutter rejection in a digital canceller, doppler filtering in a digital Fourier transform, constant false alarm rate detection in the mean-level threshold and outputting the range, azimuth, elevation, doppler and amplitude of those returns which cause threshold crossings. By data processing we mean all the information processing functions that are not covered under the headings of signal processing or radar control (which last is discussed later). This includes the functions of (1) correlation of multiple threshold crossings associated with a single target, (2) interpolation to find the position of the target in radar coordinates with the best possible accuracy, (3) decisions, according to preprogrammed rules, as to whether the observed target is the desired aircraft or, in the case of multiple targets, which one (or ones) correspond to the height request, (4) calculation of the height of the target, correcting for the earth's curvature and atmospheric refraction and (5) calculations related to the control of the radar; e.g., conversion of the height request from x-y to R- $\theta$  or calculating the maximum elevation angle required at a given range. The distinction between signal processing and data processing is largely a software distinction since both functions will be performed in the same Parallel Microprogrammable Processor (PMP) hardware.

Once the transmitted signal and the antenna scanning pattern have been determined, exact theoretical methods can be used to obtain the design of the best possible signal processor (the optimum processor). Unfortunately the optimum processor is almost never practical in terms of dollar cost and system complexity. However, the performance that would be obtained from an optimum processor can be calculated and used as a criterion for evaluating practical processors. The design of the signal processor then becomes a task of finding a non-optimum processor arrangement which offers an acceptable compromise between cost and performance. It has been found that a cascade of a three-pulse canceller and a 32-point digital Fourier transform approaches an optimum processor in performance. Use of the fast Fourier transform (FFT) algorithm makes that combination a



practical scheme in terms of system size and complexity. A magnitude approximating algorithm will be used at the outputs of the DFT and mean-level thresholds similar to those used in the first generation Moving Target Detector will follow the magnitude. The output from the signal processing module will consist of groups of digital words. These will indicate the azimuth, elevation, range, doppler frequency and amplitude of every radar return which causes a threshold to be exceeded.

In AROH the antenna elevation rate must be considerably slower than that of the unmodified FPS-6. This is so that the pulse repetition rate can be reasonably low to allow more radar pulses per 3-dB antenna beamwidth. To provide more information for determining the exact elevation of the target the elevation sweep rate will be such that there are some 54 radar pulses per 3-dB elevation beamwidth. The processor will be arranged to make the CPI's actually overlap so that two overlapping CPI's will be processed in each 48 radar pulses. This makes for a relatively slow elevation sweep rate. If the AROH always scanned up to +32 degrees like the FPS-6 does it would take too long to make a height measurement. Hence, one of the control functions in the AROH will be to limit the height of the elevation scan to the value actually required at the range of interest. This variable elevation scan will be accomplished with an electrical servo-mechanism which will replace the existing elevation drive assembly.

This limiting of the maximum elevation is one of the control functions. There are others. The basic control sequence is as follows:

- (1) Height request is received with position in X-Y coordinates and old height in feet.

- (2) AROH data processor calculates the coordinates of a volume to be searched around the point indicated in the height request. This volume nominally extends 5 nmi either side of the requested range and out 5 miles either way in the azimuth (cross range) direction. It goes from the horizon limit up to the maximum of the height coverage if there is no previous height, but may be limited to some nominal value above and below the old height if an old height is available.

- (3) The radar controller then causes the radar to search over the desired volume while the output from the signal processor is stored for final processing in the data processing module.

(4) The data processing module examines all the threshold crossings out of the signal processing module. It collects threshold crossings into groups assumed to be associated with individual targets, decides which if any is the desired target (or targets), calculates the altitude of the desired target and outputs that information for formatting and transmission to the user.

What follows this introduction is a quantitative discussion of various features of the AROH design. It is partitioned into discussions of the signal design, the radar scanning plan, modifications to the analog portions of the radar, the signal processing algorithms, the data processing algorithms, the radar control system and the interfaces with the SAGE or JSS systems.

## II. SIGNAL DESIGN

The AROH transmitted signal will be a uniform pulse train. The signals that are processed will be 32-pulse segments of this uniform pulse train.

The pulse repetition rate (or pulse repetition frequency, PRF) represents a compromise. On one side we need the PRF to be low to avoid range ambiguities and on the other we want the PRF to be high so as to provide a greater frequency spread between doppler ambiguities. In addition to these fundamental considerations the PRF is limited by FPS-6 hardware constraints.

In a height finder radar we are only interested in one target or cluster of targets at any one time, and we know the approximate range of the target a priori. Thus, it is quite conceivable, particularly in an automatic system, to optimize the PRF on a target-by-target basis. This is what is done in AROH within the limitations imposed by hardware. In AROH the PRF is always made low enough so that the desired target is in the unambiguous range interval. However, for longer range targets the PRF is made as high as possible while still meeting the above restriction. In the case of shorter target ranges, to avoid exceeding the average power capability of the transmitters, the maximum PRF is held constant at 800 pps. This, with some time allowed for recovery time, gives a maximum range of slightly less than 100 nmi.

At S-band, even with PRF's as high as 800, the unambiguous doppler range (i.e., the spacing between blind speeds) is quite limited. The spacing between blind speeds is given by the relationship:

$$V = 291 \times \frac{\text{PRF}}{f_o} \quad (1)$$

where PRF = the pulses per second

$f_o$  = the radar frequency in MHz

and V = the spacing between blind speeds in knots.

for the AROH operating at 2800 MHz it varies from 83 knots at its shorter ranges to about 41 knots at the longest range (200 nmi). We desire to provide opportunities for targets to be in those doppler filters which are clear of ground and precipitation returns. To this end, the system will operate on 2 PRF's at each range. The higher of these will be the high PRF discussed in the preceding paragraphs. It is as high as possible within limits imposed either by target range, at longer ranges, or by average power limitations at ranges less than 100 nmi. The lower PRF will be set at 80% of the higher. The radar will use the appropriate low PRF while scanning up in elevation and the higher PRF while scanning down.

### III. SIGNAL-TO-NOISE RATIO REQUIREMENT

The pulse width is reduced from 2 microseconds to 1 microsecond so that the average power capability of the system will not be exceeded. It might seem desirable to switch to a longer pulse width for longer range targets. However it will be shown here that AROH with a 1-microsecond pulse transmits adequate energy to produce a standard deviation of the height measurement of less than 1000 ft at a range of 200 nmi. It should be noted that in becoming AROH, the FPS-6 is going to get a better noise figure and a slower elevation scan rate. These two changes have the net effect of increasing the signal-to-noise ratio ( $S/N_o$ ) in the AROH by about 1 dB over that of the FPS-6. This is true even with the coaxial magnetron modification. Without that modification, assuming 5 MW can be coaxed out of the old magnetron, the signal-to-noise ratio with the AROH modifications would be about 3 dB better than that of the unmodified FPS-6.

For search radars the minimum signal-to-noise ratio required at the output of the signal processor is dictated by probability of detection and probability of false alarm requirements. Height finders are measurements radars. The requirements for signal-to-noise ratio out of height finder signal processors arise from the need for accurate angle of arrival measurements. Which is to say that the signal-to-noise ratio must be high enough to allow accurate beam splitting or angular interpolation in the elevation direction.

For AROH we have adopted the requirement that the standard deviation of that portion of the elevation error which is attributable to thermal noise and angular interpolation errors shall correspond to no more than 1000 ft of height at ranges of 200 nmi or less. This implies that at 200 nmi range the standard deviation of the elevation error must be no more than 0.047 degree. This is approximately 1/18 of the 3 dB elevation beamwidth.

The FPS-6 and the AROH get elevation information by scanning the antenna beam past the target and then making some form of interpolation or beam splitting to estimate the actual target elevation to within a small fraction of a beamwidth. This process is analogous to the search beam splitting that is done by radar video digitizers. It was analyzed years ago for the case of rapidly fluctuating targets and a non-coherent radar (P. Swerling<sup>5</sup>). That analysis is extended by Barton<sup>6</sup> to the case of slowly fluctuating targets, once again assuming a non-coherent radar processor. Significant results of these analyses are shown graphically in Figure 1 which was taken from reference 6. The AROH scans vertically at a rate of 0.75 degree every 48 radar pulses. As noted elsewhere in this report, the elevation scan rate is normalized to the radar-pulse repetition rate so that the number of pulses per 3 dB one-way elevation beamwidth (0.85 degree) is always slightly more than 54. Thus, from Figure 1 to beam split to 1/18 of the 3 dB one-way beamwidth requires a single hit signal-to-noise ratio of slightly less than 5 dB. This would be a valid estimate of the AROH signal-to-noise requirement if AROH were a non-coherent radar using a simple center of gravity estimate for its beam splitting. However, AROH is a coherent system and will use a more nearly optimal processor for angle estimation. Hence, the conclusion that the AROH will provide the desired elevation accuracy with a single hit signal-to-noise ratio of 5 dB is conservative.

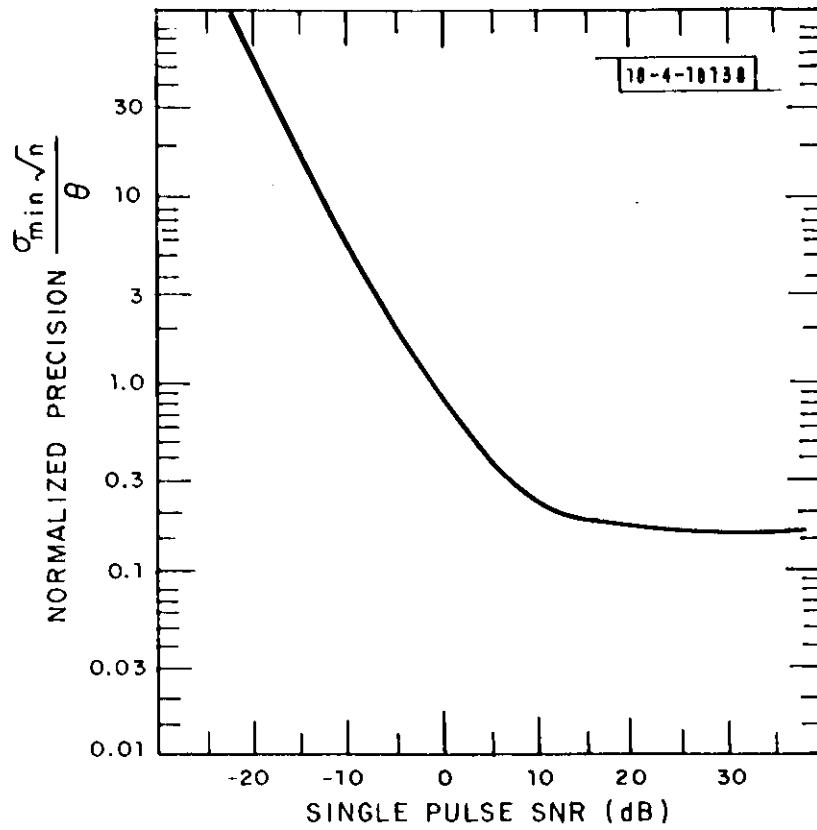


Fig. 1. Angular Precision for Scanning Radar.

Some useful perspective may be gained by applying the same criterion to the unmodified FPS-6. At long ranges and reasonably low target heights, the FPS-6 produces on the order of 20 hits per beamwidth. Thus the signal-to-noise ratio required per hit to provide beam splitting to 1/18 of a beamwidth would be on the order of 10 dB.

For a constant height accuracy the signal-to-noise ratio required increases as the square of the radar range for all but very short ranges. This is because the angle subtended by a constant height error decreases linearly with range and the signal-to-noise required varies inversely as the square of the required standard deviation in angle error. This  $R^2$  law compounds the normal  $R^4$  law associated with radars. Hence, extending the range of a height finding system while maintaining such things as scan rate constant requires that the transmitter power be increased as the 6th power of the range. For example, to increase the range of AROH from 200 to 250 nmi while maintaining the same data rates etc. would require increasing the transmitted power by a factor of approximately 3.8.

#### IV. SIGNAL-TO-NOISE CALCULATIONS

In this section we use the radar range equation to predict the signal-to-noise ratios that will be experienced by the AROH and contrast them with similarly calculated values for the unmodified FPS-6. The form of the radar range equation used is given below.

$$\frac{S}{N} = \frac{\hat{P}_t \tau G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 (kt) F_n L} \quad (2)$$

where

$\frac{S}{N}$  = signal-to-noise ratio

$\hat{P}_t$  = peak transmitted power

$\tau$  = transmitted pulse width

$G$  = the antenna gain (estimated here from the formula  $G = \frac{33000}{\theta_a \theta_e}$  where  $\theta$ 's are one-way 3 dB beamwidths

$\lambda$  = wavelength  
 $\sigma$  = target cross section  
R = target range  
K = Boltzmann's constant  
T = 290K  
 $F_n$  = receiver noise figure  
L = total of system losses

The radar range equation can be a pernicious misguider of the unwary if the value assigned to losses is not selected with considerable care. Accordingly, the various system losses are discussed below.

Certain loss mechanisms are active in all radar systems. These include the following:

Propagation - Height finders, because of their relatively narrow vertical beamwidths, are free of the ground reflection problems that bother some other types of radars. However, at E-band atmospheric attenuation can be significant at longer ranges. For this study we have selected a value of 3 dB for the two-way attenuation over a 200-nmi path at the elevation angles normally used by a height finder<sup>7</sup>.

Antenna beam shape - Clearly the antenna gain will be reduced from its nominal value if the nose of the beam is not pointed directly at the target. The antenna can be pointed off in either azimuth or elevation. For the FPS-6 we assume that the operator has zeroed in on the azimuth. However, the AROH must scan some predetermined azimuth sector, hence is penalized with a scanning loss in azimuth. The AROH azimuth scan is arranged so that it makes one complete nodding cycle as the azimuth moves through a beamwidth. Thus there are two looks at the target as the antenna scans through a 3 dB one-way beamwidth in azimuth (one look while nodding up and the second while nodding down or vice versa). Thus, if we ignore blind speeds for the moment, the worst situation is when the two nodes exactly straddle the nose of the beam. In this case the data processor can average the height measurements from two statistically independent looks at the target. Each of

the two looks will have been made with a signal-to-noise ratio that is down by 1.4 dB. Averaging over these two results in a height estimate which is slightly more accurate than would be obtained from a single look on the nose of the azimuth beam. Hence, except for the effects of blind speeds, there is essentially no loss due to azimuth off-boresight conditions.

As will be discussed in a later section, only three of the 32 doppler channels will be affected by blind speeds. Further, since two pulse repetition rates are used on alternate nods, the blind speed for any given target will only affect every other nod. Hence, the net effect of a blind speed will be to reduce the number of effective nods from two per azimuth beamwidth to one. This will make the worst case straddle that in which the data processor must average two height estimates obtained with signal-to-noise ratio down 6 dB from the nominal. This corresponds to a net loss of 3 dB. (The best situation would be when the blind nods straddled the azimuth beam and the processor got one estimate from the nose of the azimuth, i.e. no loss.) We have assume that when averaged over all possibilities the average loss due to blind speeds is on the order of 1.6 dB. To be conservative we assume that, in actual operation, some of the targets among the 10 percent that fall into blind speed zones may well be very important ones. Accordingly, we put the loss of 1.6 dB into the range equation to account for that situation.

Nodding beam height finders are similar to scanning search radars in that the antenna is continually being scanned past the targets in one dimension (i.e., elevation for the height finder and azimuth for the search radar). In considering probability of detection in search radars it is common practice to expect a loss, usually 1.6 dB, because of the effects of antenna beam shape as the antenna scans by the target. This particular loss has been assimilated into the analyses of angular accuracies used in this report. Hence, it need not be considered in the radar range equation.

Plumbing - It is well known that radar signals are attenuated in the waveguide and other microwave transmission hardware. In AROH on a 50-ft tower the theoretical two-way loss in the waveguide will be about 1.5 dB. More or less arbitrarily we assume another 2.5 dB in the other microwave gadgetry for a total two-way plumbing loss of 4 dB.



Field Degradation - Earlier radars used vacuum tubes throughout except for relatively short lived point contact diodes in the signal mixers. It was conventional to assign 2 or 3 dB of loss to account for the unreliability and drift that were experienced in the vacuum tube receivers along with the relatively short life of some other components and the often inept maintenance that occurred. AROH, like other MTD radars and most modern radars, will be built from contemporary solid-state components. These components have proved themselves to be both reliable and stable. Hence, we are only assuming one dB of field degradation loss for the AROH.

Pulse waveform mismatch - While the theory of matched filtering is well known a simple radar with a filter exactly matched to the received pulse is seldom found in practice.

The transmitted waveform is almost always a simple rectangular pulse. In the receiver it is filtered using a filter whose bandwidth is closely related to the reciprocal of the pulse width. The losses to be expected from various filters are shown in Skolnik's handbook<sup>7</sup> (page 2-15). A reasonable loss for a well matched filter is 0.5 dB.

The loss mechanisms discussed above may be found in any surveillance radar. There are additional loss mechanisms which are specific to range gated pulsed doppler radars. To wit:

Range Gate Straddling - Range gate sampling in AROH occurs at equally spaced points in time. The target returns, however, appear at any point in time and the maximum output of the pulse filter will in general not occur at the range gate sampling time. In search radars where the signal-to-noise ratios are relatively low, losses due to this range gate straddling can be significant. However, when the objective of the radar is measurement rather than detection the loss due to gate straddling can usually be recovered by smoothing over the resulting multiple data points. In our case, a 1-microsecond pulse is used and sampling occurs at 0.77 microsecond intervals. Thus an equally split target would be sampled at  $\pm 0.385$  microsecond. Skolnik<sup>7</sup> (page 5-26) shows a signal reduction of about 3 dB at this point for reasonable filter shapes. Hence the standard deviation of the

angle error from one range gate would increase to 1.41 of the nominal error. However the processor can smooth over two such estimates reducing the error thereby to 0.707 of the error from either one. Thus in AROH we need not take a loss from range gate splitting.

Doppler Weighting - In doppler filter banks derived from digital Fourier transforms weighting is used to reduce the doppler sidebands. This weighting on the input signals also causes an increase in the effective filter bandwidth. This in turn lets more noise through than a perfectly matched filter would. In the AROH processor most of the doppler side lobes are held below 40 dB down in this way. The loss due to this weighting is on the order of 1.6 dB.

Doppler Filter Straddling - A loss can occur because the target may have a doppler offset which does not fall on the peak of any filter response. As in range gate straddling, this loss can be significant for search radars where the objective is target detection at relatively low signal-to-noise ratios. However, in measuring elevation angles the split gives rise to a pair of statistically independent estimates which can be smoothed to essentially eliminate the loss due to doppler filter straddling.

The loss mechanisms that may be included in various signal-to-noise calculations in this report are summarized in the following table:

<u>Mechanism</u>	<u>Loss (dB)</u>
Atmospheric absorption	3
Beam shape (azimuth)	1.6
Plumbing	4
Pulse waveform mismatch	0.5
Field degradation	1.0
Doppler weighting*	1.6

\*Does not apply to single pulse calculations.

Thus the total loss allowance for single hit calculations is 10.1 dB. Using that value along with the parameters listed in Table 1 in the radar range equation we find that the AROH may be expected to produce a single pulse signal-to-noise ratio of about 5 dB on a 5-square-meter target at a range of 200 nmi.

TABLE 1  
RADAR PARAMETERS

<u>Parameter</u>	<u>AROH</u>	<u>FPS-6</u>
Peak power	3 MW	5 MW
Pulse width	1 $\mu$ sec	2 $\mu$ sec
Antenna		
Elevation Beamwidth	0.85 <sup>o</sup>	0.85 <sup>o</sup>
Azimuth Beamwidth	3.2 <sup>o</sup>	3.2 <sup>o</sup>
Gain	41 dB	41 dB
Hits/Beamwidth	54	10 to 20
Elevation scan	variable	-2 <sup>o</sup> to +32 <sup>o</sup>
Noise figure	7.5 dB	9 dB

A similar calculation shows that the FPS-6 might be expected to produce a single pulse signal-to-noise ratio of slightly less than 9 dB which, from the previous discussion, is about 1 dB less than that required to produce a 1000-ft height accuracy at 200 nmi from a 5-square-meter target.

In passing through the signal processor the radar echoes are integrated in groups of 32. If the processor were perfect we would expect the signal-to-noise ratio to be improved by a factor of 32 (approximately 15 dB). However, the signal processor incurs a doppler filter weighting loss of 1.6 dB so that the net improvement is only 13.5 dB. In calculating the signal-to-noise ratio out of the signal processor we include the azimuth beam shape loss as before. The calculation is for reference purposes in simulating various azimuth interpolating algorithms so we do not include any elevation beam shape loss. Thus, if the antenna were stopped pointing directly at the target we would expect the signal-to-noise ratio at the processor output to be 18.4 dB for a 5-square-meter target at 200 nmi.

As noted earlier, the signal-to-noise ratio required for a constant rms height error varies approximately as the square of the radar range. The approximation is conservative in the sense that at higher elevations where it breaks down it calls for more signal-to-noise ratio than is actually required. It is accurate to within 1 dB for elevation angles up to 26 degrees. The signal to noise versus

range requirement for AROH, approximated this way, is plotted on Figure 2. Clearly, for targets in clear areas the signal-to-noise ratio available to the radar increases as the inverse fourth power of the radar range. However the requirement of Figure 2 will be useful in considering the AROH performance in areas of precipitation or ground clutter.

#### V. PRECIPITATION CLUTTER

The AROH system, if it is to be truly automatic, must cope automatically with precipitation clutter. Ideally, it would provide the same probability of an accurate height readout in widespread precipitation conditions as it does when there is no clutter. At the very worst, it must not produce false height readouts from precipitation clutter returns. In theory it is possible to design a system which approaches the ideal. Actual performance of the AROH system, which is constrained to the frequency and PRF limitations of the FPS-6, is expected to fall somewhere between these two extremes.

Precipitation is a volume scatterer. At any one time the radar will be receiving energy reflected from the contents of an entire range-azimuth-elevation resolution cell. The volume of the resolution cell is given by:

$$V = R^2 \theta \phi \Delta R \quad (3)$$

where: R is the range  
 $\theta$  is the two-way azimuth half power beamwidth in radians  
 $\phi$  is the two-way elevation half power beamwidth in radians  
 $\Delta R$  is the length of the range resolution cell (150 meters for a 1  $\mu$ sec pulse)

for the FPS-6 antenna, 1  $\mu$ sec pulse and range in nmi.

$$V = 2.09 \times 10^5 R^2 \quad (4)$$

To find the effective cross section of a resolution cell full of rain we need to know the volume reflectivity of the rain in the cell. There appears to have been good agreement on this subject for a number of years. Nathanson gives the following figures in his recent book<sup>3</sup> for rain returns at the top of E-band.

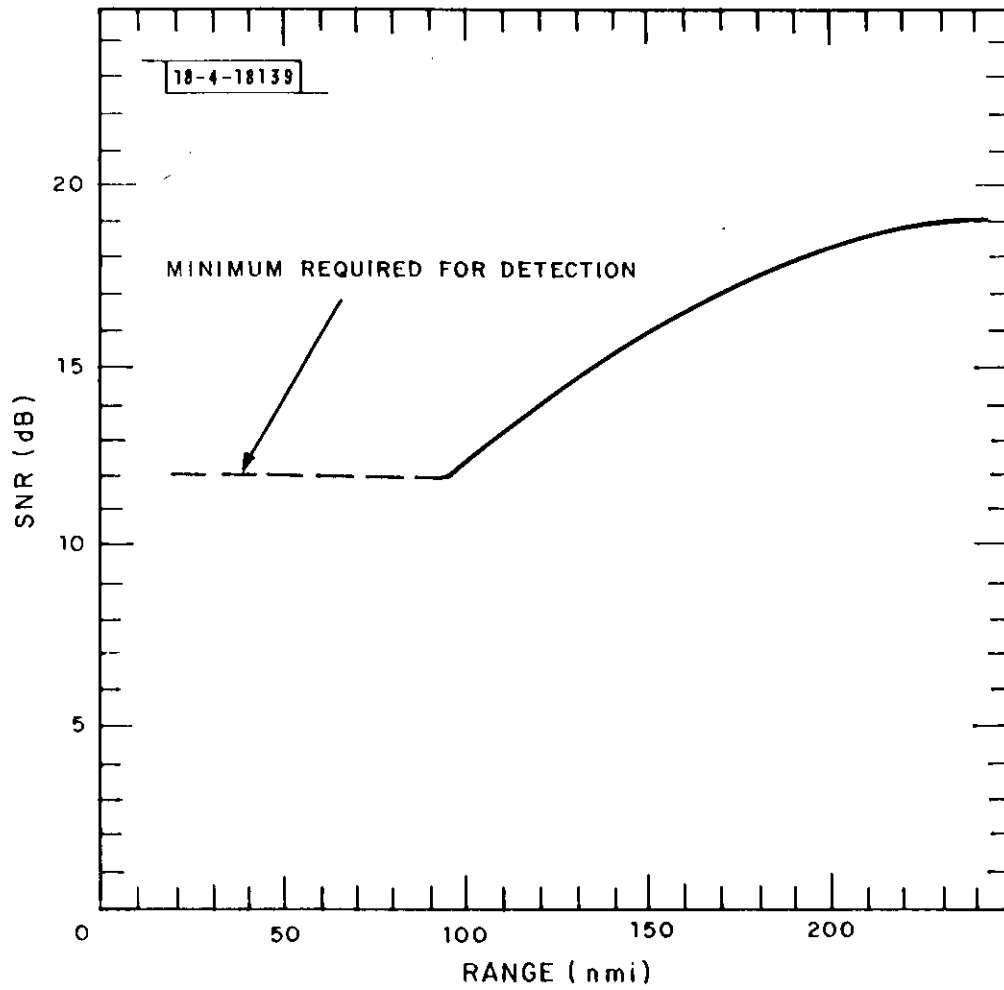


Fig. 2. AROH Signal-to-Noise Ratio per CPI Required for  $1000^{\circ}$  RMS Height Error.

<u>Type</u>	<u>Probability of Occurrence Washington, DC (percent)</u>	<u>Reflectivity (dB) above 1-sq. meter per cubic meter)</u>
Light rain 1 mm/hr	3	-92
Moderate rain 4 mm/hr	0.7	-83
Heavy rain 16 mm/hr	0.1*	-73

\*0.1 percent of the time amounts to less than 9 hours per year.  
 These numbers agree well with those reported earlier<sup>9,10</sup>.

The rain cross section seen by the radar is the product of the resolution cell volume and the reflectivity for the particular conditions. It is plotted for the AROH system in Figure 3. Widespread rain (i.e., rain which is not associated with convective storms) usually extends from the ground up to some ceiling between about 10,000 ft and 20,000 ft. Hence, at the longer ranges most of the precipitation may well be below the radar horizon. This is indicated by the dashed portion of the curves in Figure 3. It can be seen that, except for moderate rain at the very shorter ranges, the signal processor must effect considerable improvement in the target-to-rain-clutter ratio if the AROH is to be effective.

Rain and snow while falling are blown about by the wind. The radial component of that motion imparts a doppler shift to the radar returns. The resulting doppler spectrum is spread by a number of effects. The most important of these for the AROH are the effects of wind shear and turbulence. The following discussions of the precipitation doppler spectrum is adapted from that of Nathanson<sup>11</sup>.

We assume that the wind shear situation can be approximated by a constant gradient of wind speed vs. altitude. This situation is depicted in Figure 4. For elevation angles of a few degrees the difference in the radial velocity across the beam is

$$\Delta V_R = |V_{R1} - V_{R2}| \quad (5)$$

If we assume a Gaussian antenna pattern and a constant velocity gradient across the beam, the velocity distribution will have a standard deviation given by

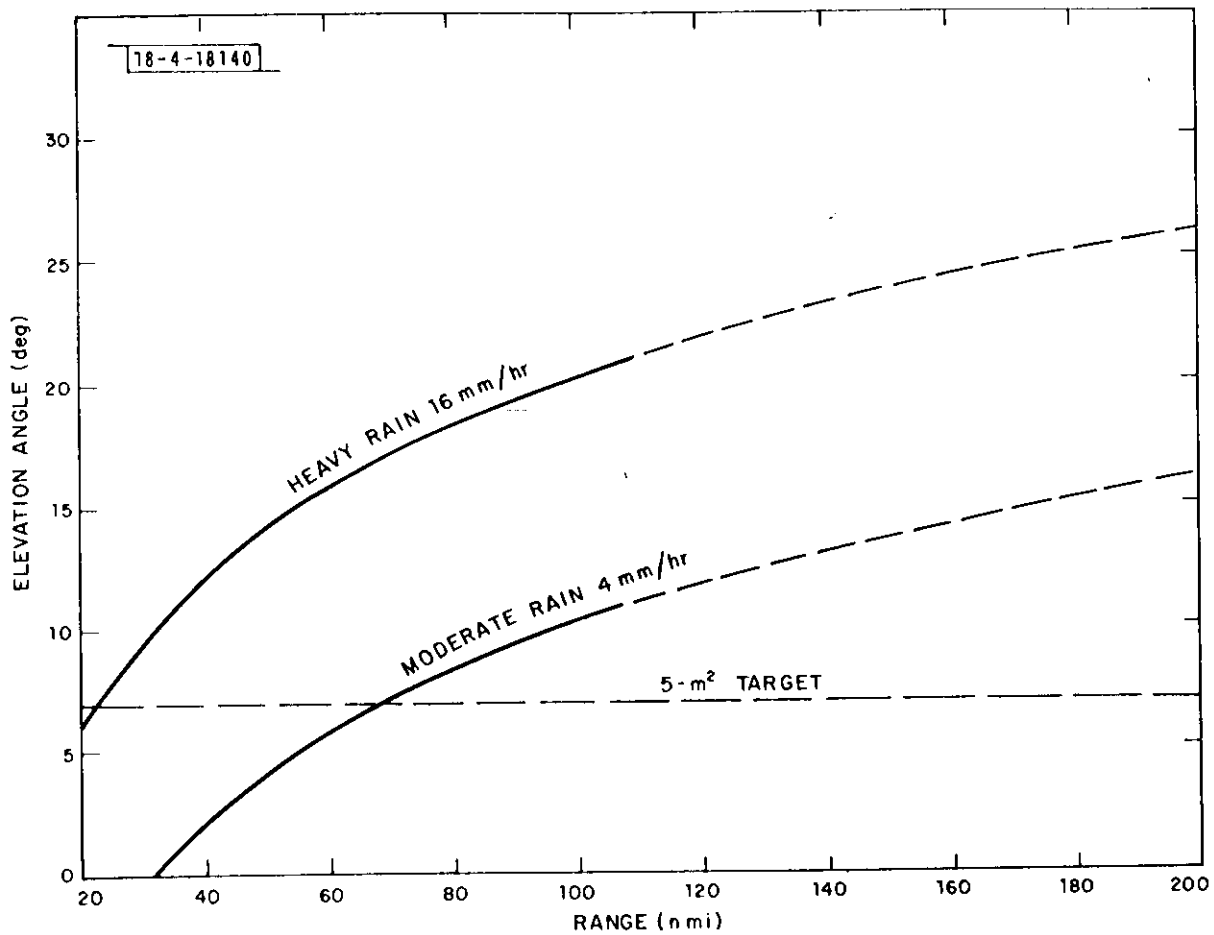


Fig. 3. AROH Effective Rain Cross Section.

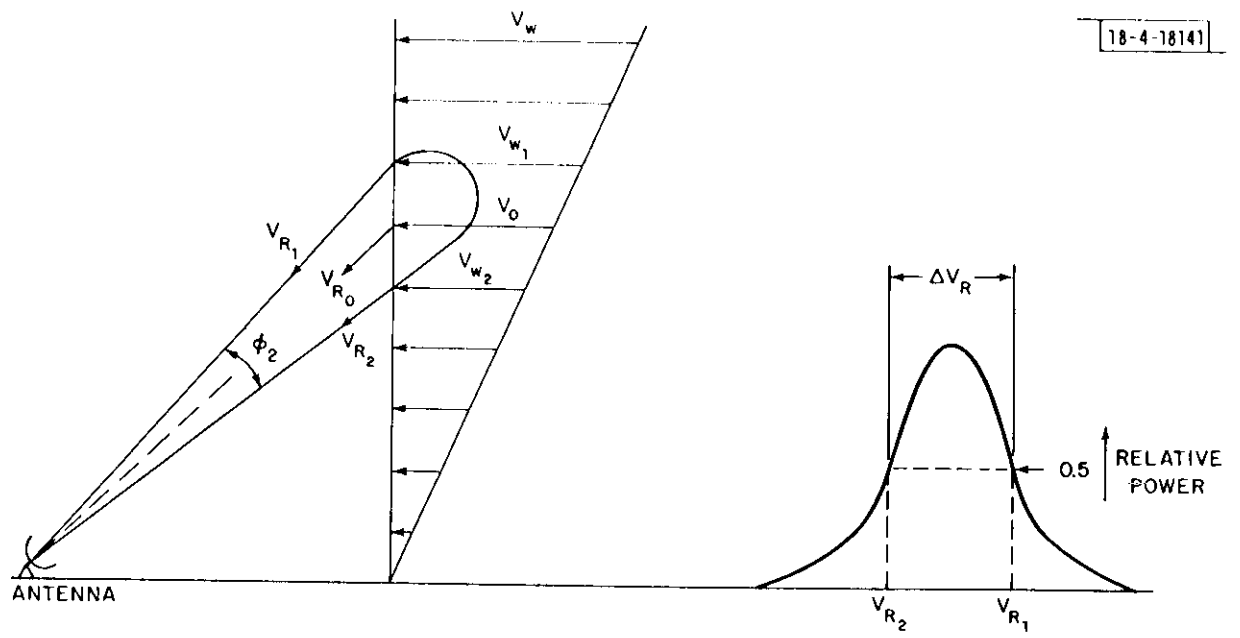


Fig. 4. Effects of Wind Shear on the Doppler Spectrum.



$$\begin{aligned}\sigma_{\text{shear}} &= 0.42 (\Delta V_R) \\ &= 0.42 k R \sin \phi_2\end{aligned}\tag{6}$$

where  $K$  = the velocity gradient along the beam,  $R$  is the slant range to the resolution cell in question and  $\phi_2$  is the two-way half power elevation beamwidth. For  $R$  in nmi and  $\sigma_{\text{shear}}$  in knots a conservative value for  $k$  is 20.5.

Wind velocity is not constant with time. Hence, the mean wind velocity is only defined vigorously when an averaging time is specified. Fluctuations about the mean may be called turbulence. These fluctuations are unpredictable and must be described statistically. Nathanson gives an average value for the standard deviation of variations due to turbulence at low and moderate altitudes as one meter per second (slightly less than 2 knots). There can also be spectral spreading due to the horizontal beamwidth spanning different velocities and to the statistical nature of the fall velocities of the precipitation. These latter two mechanisms turn out to be unimportant in the AROH case.

Thus, we have

$$\sigma_v = \sqrt{(\sigma_{\text{shear}})^2 + 4}\tag{7}$$

where  $\sigma_{\text{shear}} = .09 R$  and  $R$  is the range in nmi,  $\sigma_v$  is the width of the radial velocity spectrum from the mean radial velocity to the half power points. We define the width of the clutter region in the radial velocity dimension as the space between the  $2 \sigma_v$  points of the spectrum, thus

$$\Delta V = 4 \sigma_v\tag{8}$$

The estimated width of this cluttered region is plotted as a function of range in Figure 5. These estimates are consistent with our experience using the MTD with E-band ASR radars.

As is well known, the radial component of the target or clutter velocity manifests itself at the radar as a doppler offset of the frequency of the radar return. If the target or clutter is approaching the radar the frequency of the radar echo will be slightly higher than the transmitted frequency and, conversely, if the target is receding from the radar, the echo frequency will be lower.

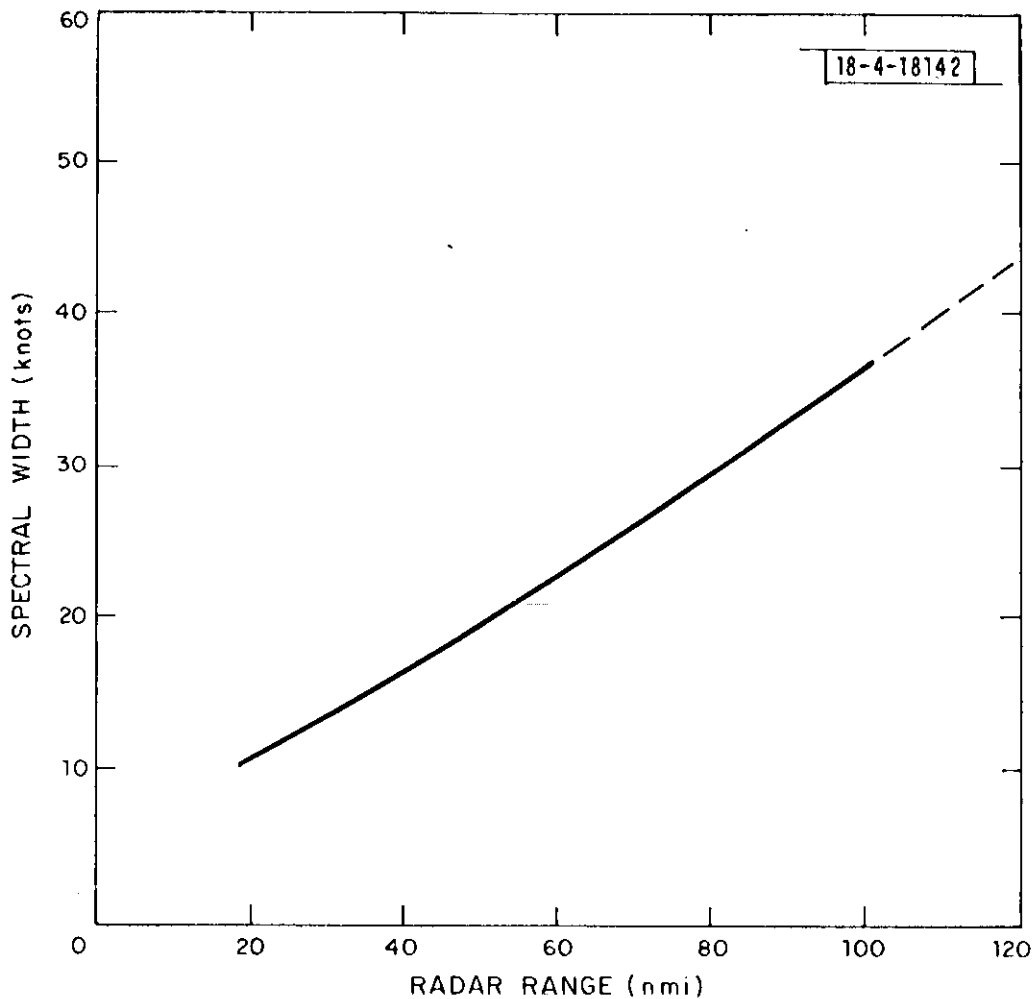


Fig. 5. Estimated Rain Return Spectral Width.

The magnitude of this doppler offset is given quite accurately for relatively narrow-band systems like AROH by the approximation:

$$f_d \approx \frac{2 V_r}{\lambda} \quad (9)$$

where  $f_d$  is the doppler offset in the Hz

$V_r$  is the radial velocity in meters/sec, and

$\lambda$  is the wavelength in meters

For a radar frequency of 2800 MHz and radial velocities in knots, the above equation becomes

$$f_d = 9.6 V_r \quad (10)$$

In AROH, as in all pulse doppler radars there will be doppler ambiguities. These ambiguities are well known as blind speeds in conventional MTI radars. Blind speeds are known to occur when the target's doppler offset is equal to the radar pulse repetition frequency. Thus, for AROH operating at a radar frequency of 2800 MHz and a pulse repetition rate of 800, the first blind speed is approximately 83 knots. This means that a target with a radial velocity component of any multiple of 83 knots will look to the radar exactly like a stationary target. Similarly, a target moving radially at 93 knots (or 176 knots or 259 knots, etc.) will look to the radar exactly like a target moving radially at 10 knots.

More precisely, a set of unambiguous doppler frequencies is defined by the inequality relation;

$$|f| \leq \frac{\text{PRF}}{2} \quad (11)$$

All radar returns having doppler frequencies outside that range will be aliased so as to have the apparent doppler frequency which satisfies the relationships

$$f_a = f_d + K (\text{PRF}) \quad (12)$$

with K an integer such that

$$|f_a| \leq \frac{\text{PRF}}{2}$$

Thus, all the radar returns observed by AROH appear to have radial velocities of about 41 1/2 knots or less.

The signal processor in AROH will divide the unambiguous doppler range up into 32 filter outputs. All targets will appear in one or a few of these filters. Similarly, precipitation returns will appear in anywhere from four to as many as 16 or more of the 32 filter outputs. Those targets which happen to fall in one of the filters containing precipitation returns must be significantly stronger than the weather to be processed successfully. However, those radar targets which appear in filters that do not contain weather return can be processed successfully even if their radar cross section is considerably smaller than that of the weather.

It will be recalled that the AROH uses pairs of pulse repetition frequencies. The aircraft doppler frequency folds over (aliases) differently on the different PRF's so that the return from an aircraft with a radial velocity outside the unambiguous doppler range will appear in different sets of doppler filters in the different pulse repetition frequencies. The weather returns, on the other hand, are at relatively low velocities and will appear at nearly the same doppler frequency on both pulse repetition rates. Thus, many targets which are competing with weather returns at one pulse repetition rate will be in the clear at the other. From the above it can be seen that although the AROH does not attain the ideal of no degradation at all in precipitation, it provides a considerable improvement over the non-coherent FPS-6. Still better subweather performance could be had if the pulse repetition rate could be higher or if the radar frequency could be lower, however these parameters are restricted to those available from the FPS-6 radar.

## VI. GROUND CLUTTER

The semiautomatic system using the FPS-6 relied on the skill and wisdom of the operator to reject ground clutter and to detect low flying targets in the interstices between clutter areas. The operator will be lost to the automatic system and the burden of dealing with ground clutter will fall on the radar and digital processing system. In addition to detecting targets over ground clutter, the system must reduce to an acceptable level the height errors caused by ground clutter residue. This imposes stringent requirements on the radar and the signal processor.

It is difficult if not impossible to define a standard for clutter rejection that will insure satisfactory operation of the AROH system at all sites and at minimum hardware cost. Different sites vary widely in the area and intensity of clutter return. Almost all sites will have a few range azimuth cells at which the ground clutter is so strong that low aircraft height accuracy will be degraded. The problem of deciding what is acceptable performance comes down to that of deciding how many range azimuth cells may be lost at the low elevations before the performance of the system is declared to be unacceptable. The AROH system as presently planned will have about half a million range azimuth resolution cells. An approach to specifying a clutter rejection requirement might be to decide from operational requirements how many of the half million cells with degraded low altitude performance can be tolerated. The clutter statistics of all the sites could be surveyed or estimated. Appropriate requirements would then be specified for the radar and processor.

As an alternative we have elected to design a system which is as close to the state of the art as the resources of the project will allow. We expect to show that such a design will be operationally quite satisfactory and will in fact provide significantly better low altitude coverage than any radar presently in the air defense system.

A prerequisite for accurate height measurements on moving targets in the presence of fixed clutter is adequate linear dynamic range. If the ground clutter causes gain compression in the receiver (or A/D converter or signal processor), errors will be introduced into the elevation interpolation. The ultimate limitation on the dynamic range appears to be the analog-to-digital converters. State-of-the-art A/D converters deliver 10-bit precision at rates of 3 million conversions per second. These are used in AROH to convert analog bipolar video signals to digital numbers containing nine digits and a sign bit. The largest clutter signal that the system can handle is one that just fails to cause overflow of the A/D converter. Similarly the smallest signals that will be observed are signals that cause changes of at least one least significant bit in the A/D converter. Thus, with a nine-bit plus sign converter the theoretical linear dynamic range from the smallest signal to peak clutter will be 54 dB. In a practical system it is

unlikely that the system would be adjusted perfectly for all situations. Hence, it seems more realistic to expect operation at clutter-to-target ratios up to a maximum of about 50 dB. This should be compared with existing search radars which can handle maximum clutter-to-target ratios on the order of 25 to 30 dB.

Ground clutter is usually considered as being either continuous or discrete. Continuous clutter is characterized by a dimensionless quantity  $\sigma_o$  which relates the radar target cross section of distributed clutter to the area mapped out on the ground by the radar's range-azimuth resolution cell. Thus the effective clutter cross section is a function of the range and of the radar parameters:

$$\sigma = \sigma_o R \theta \Delta R \quad (13)$$

where  $\sigma$  = the clutter backscatter cross section  
 $\sigma_o$  = the clutter backscatter coefficient  
 $R$  = the radar range  
 $\theta$  = the two-way 3-dB beamwidth (in radians)  
 $\Delta R$  = the range resolution cell

For AROH this becomes approximately

$$\sigma = 10,860 R \sigma_o \quad (14)$$

for  $R$  in nmi and  $\sigma$  in square meters.

There is more literature than agreement on values to expect in various situations, however we feel conservative in demanding that AROH operate adequately at sites surrounded by extensive areas where the ground clutter  $\sigma_o$  is as great as -10 dB. This is a value which was reported as being exceeded less than one percent of the time by an E-band radar with a resolution cell size similar to the AROH sited in the Rocky Mountains<sup>8</sup>. With a  $\sigma_o$  of -10 dB, AROH will operate properly against a 1-square-meter target at a range of almost 100 nmi.

Discrete scatterers, as the name implies, are isolated objects which cause large radar returns. Examples are large buildings, television or microwave towers and the like. Most radars sited in or near populated areas get returns from a number of such discrete scatterers. These scatterers are characterized by a radar cross section. At most sites there are only a few tens of discrete scatterers

with radar cross sections exceeding 100,000 sq meters<sup>13</sup>. Hence, we expect that a system which can operate with a 50-dB clutter-to-target ratio will provide adequate coverage.

MTD processor designs for search radars usually include some form of automatic clutter map<sup>12</sup>. The map stores the magnitude of the ground clutter return in each range azimuth cell. The value in each cell is updated on each scan of the radar. This clutter amplitude map is used to set a zero velocity threshold for each range azimuth cell. Thus the MTD processor with the map can detect targets with zero radial velocity if the target amplitude is sufficiently greater than the clutter amplitude in that particular range cell.

The use of such a scheme was considered for AROH. However, AROH would require a three-dimensional clutter map rather than the simpler two-dimensional map required by search radars. Hence a map for AROH would require an order of magnitude more storage. Further the AROH would need more precision in the stored values than is required for search radars. Note that in the case of the scanning two-dimensional search radar, periodic updating of the clutter map is a simple matter. By contrast, the AROH does not scan uniformly. In AROH updating of the clutter map would have to be programmed as a low priority background sort of activity. Finally, the zero velocity notch in AROH is only about 10 knots wide as opposed to about 30 knots in the ASR's. Consideration of all these factors led to the conclusion that the clutter map was not a cost effective scheme for use with AROH. Hence, in this first generation AROH there will be no attempt to detect truly tangential targets.

## VII. DOPPLER FILTERS

In the design of a digital signal processor for a radar in an automatic system, one must get the best possible signal-to-clutter improvement that is consistent with a reasonable amount of complexity. Rigorous theoretical methods are available for designing signals and processors which are optimized for rejecting a combination of ground clutter return and thermal noise (reference 14). This theory, which is outlined below, provides the design of a processor which optimizes the rejection of ground clutter but which is in general costly to implement.

Thus implementing a set of optimal filters for a 32-pulse coherent processing interval would require 1028 complex multiplies per range gate. By contrast a 32-point fast Fourier transform (FFT) filter set requires only 80 complex multiplies. Further, the optimum filter which is optimized only for ground clutter and noise may not work as well as a good suboptimum filter when precipitation clutter is present. In general, the optimum filters have relatively high doppler side lobes. A good suboptimal filter may have almost as good ground clutter rejection as an optimal filter and have much better doppler side lobes. Low side lobes over the entire unambiguous doppler region are required for good rejection of precipitation clutter.

It is convenient to know the performance of the optimum filter set. Then once a set of non-optimum filters with low side lobes is designed, the clutter rejecting performance of those filters can be compared to the performance of the optimum set.

In the following paragraphs the theory of the optimum filters will be reviewed and a set of non-optimum filters which seem appropriate for AROH will be presented.

#### A. N Pulse Processor Theory

Consider the return of N equally spaced radar pulses from a target and from ground return. The target signal return may be represented by the vector

$$S^* = [S(\theta_1), S(\theta_2) e^{-i\omega T}, S(\theta_3) e^{-i2\omega T}, \dots, S(\theta_N) e^{-i(n-1)\omega T}] \quad (15)$$

where  $S(\theta_n)$  are weights generated by the two-way antenna pattern  $S(\theta)$ .  $\theta_n$  is the angle between boresight and the target during the nth pulse.  $\omega$  is the angular doppler frequency of the target,  $T$  is the radar interpulse period and  $*$  represents the conjugate transpose. The vector elements are normalized so that there is unity power into the filter.

$$S^* S = 1 \quad (16)$$

In a similar manner the clutter return vector (which is random) is represented by

$$\xi^* = [\xi_1, \xi_2, \xi_3, \dots, \xi_N] \quad (17)$$



where  $\xi$  are complex random variables with the following properties. The expectation of the product of the  $n$ th and the conjugate of the  $n+r$  clutter return is

$$E \{ \xi_n \bar{\xi}_{n+r} \} = \sigma_r \quad (18)$$

where  $\sigma_r$  by definition is the covariance of the clutter (including noise) at the different  $t = rT$ .  $\sigma_r$  is calculated from the beam pattern and rotational motion of the antenna<sup>15</sup>.  $\xi$  is normalized as before so that there is unity clutter power into the filter, i.e.

$$E \{ \xi^* \xi \} = 1 \quad (19)$$

The processor ("optimum" or not) multiplies the incoming  $N$  pulses by a set of weights.

$$W^* = [W_1, W_2, W_3 \dots W_N] \quad (20)$$

Thus the signal power out of the processor is given in terms of the inner vector product.

$$S_{out} = |W^* S|^2 = (W^* S) (S^* W) \quad (21)$$

and the average clutter out of the processor is

$$\begin{aligned} C_{out} &= E \{ W^* \xi \xi^* W \} \\ &= W^* E \{ \xi \xi^* \} W \\ &= W^* M W \end{aligned} \quad (22)$$

where the expectation of the outer product is by definition the clutter covariance matrix  $M$ .

Since the input signal and clutter powers have been set to 1, the filter output signal-to-clutter ratio  $S_{out}/C_{out}$  is the improvement factor for a particular set of weights  $W$ . The improvement is therefore

$$\beta(\omega) = \frac{|W^* S|^2}{W^* M W} = \frac{(W^* S) (S^* W)}{W^* M W} \quad (23)$$

$\beta$  is a function of  $\omega$  because  $S = S(\omega)$  (cf. Eq. 15);  $\beta$  is always real and non-negative.

Thus implementing a set of optimal filters for a 32-pulse coherent processing interval would require 1028 complex multiplies per range gate. By contrast a 32-point fast Fourier transform (FFT) filter set requires only 80 complex multiplies. Further, the optimum filter which is optimized only for ground clutter and noise may not work as well as a good suboptimum filter when precipitation clutter is present. In general, the optimum filters have relatively high doppler side lobes. A good suboptimal filter may have almost as good ground clutter rejection as an optimal filter and have much better doppler side lobes. Low side lobes over the entire unambiguous doppler region are required for good rejection of precipitation clutter.

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The processor ("optimum" or not) multiplies the incoming  $N$  pulses by a set of weights.

$$W^* = [W_1, W_2, W_3 \dots W_N] \quad (20)$$

Thus the signal power out of the processor is given in terms of the inner vector product.

$$S_{out} = |W^* S|^2 = (W^* S) (S^* W) \quad (21)$$

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$$\begin{aligned} C_{out} &= E \{ W^* \xi \xi^* W \} \\ &= W^* E \{ \xi \xi^* \} W \\ &= W^* M W \end{aligned} \quad (22)$$

where the expectation of the outer product is by definition the clutter covariance matrix  $M$ .

Since the input signal and clutter powers have been set to 1, the filter output signal-to-clutter ratio  $S_{out}/C_{out}$  is the improvement factor for a particular set of weights  $W$ . The improvement is therefore

$$\beta(\omega) = \frac{|W^* S|^2}{W^* M W} = \frac{(W^* S) (S^* W)}{W^* M W} \quad (23)$$

$\beta$  is a function of  $\omega$  because  $S = S(\omega)$  (cf. Eq. 15).  $\beta$  is always real and non-negative.

The "optimum" processor is the processor whose weights  $W$  maximize  $\beta$  in Eq. (23).  $M$  is a positive definite matrix which possesses a square root and inverses, i.e.

$$M = M^{1/2} M^{1/2}$$

$$M^{-1} M = I$$

and

$$M^{1/2} M^{-1/2} = I$$

where  $I$  is the unity matrix. Thus we can rewrite Eq. (23).

$$\beta = \frac{(W^* S) (S^* W)}{W^* M W} = \frac{(W^* M^{1/2} M^{-1/2} S) (S^* M^{-1/2} M^{1/2} W)}{W^* M W}$$

and by the Cauchy-Schwartz Inequality

$$\beta \leq \frac{(W^* M^{1/2} M^{1/2} W) (S^* M^{-1/2} M^{-1/2} S)}{W^* M W} \quad (24)$$

Thus the maximum or "optimum" value of  $\beta$  occurs when the equality exists:

$$\beta_{\text{opt}} = S^* M^{-1} S$$

The optimum set of weights are found when the appropriate  $W$  is substituted in Eq. (23) and the result equals

$$\beta = \beta_{\text{opt}} = S^* M^{-1} S$$

This occurs when

$$W_{\text{opt}} = k M^{-1} S \quad (26)$$

where  $k$  is an arbitrary (complex) constant.

It should be noted that in Eq. (25) the improvement factor is a function of the target doppler frequency  $\omega$ , and that as  $\omega$  varies so do the optimum weights,  $W_{\text{opt}}$ . Thus, Eq. (25) represents the locus of the infinite set of improvement factors, that is

$$\beta_{\text{opt}}(\omega) = S(\omega) M^{-1} S(\omega) \quad (27)$$

This curve is shown in Figure 6. The covariance matrix  $M$  was calculated assuming the following:

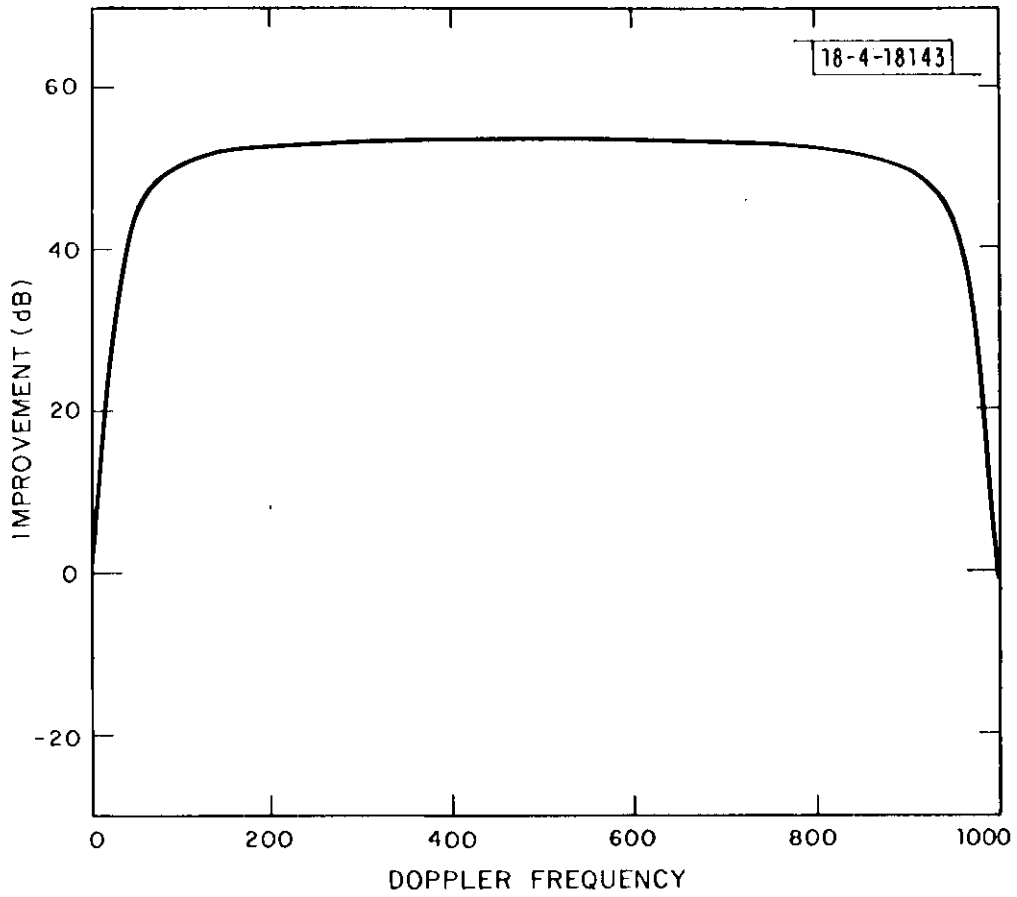


Fig. 6. Locus of Optimum Improvement Factors.

1. Antenna scan rate of 60.7 pulses/degree
2. A  $.85^\circ$  beamwidth antenna having a cosine amplitude distribution
3. A 40 dB C/N ground clutter to thermal noise ratio.
4.  $N = 32$  pulses are processed.

In general, one selects approximately  $N$  equally spaced filters at frequencies  $\omega_n$  thus fixing  $W_{\text{opt}}$  to  $N$  specific vectors using Eq. (26)

$$W_{\text{nopt}} = kM^{-1}S(\omega_n) \quad n = 1, 2, \dots, N \quad (28)$$

These weight vectors  $W_{\text{nopt}}$  are then used in Eq. (9), i.e.

$$\beta_n(\delta) = \frac{|W_{\text{nopt}}^* S(\omega)|^2}{W_{\text{nopt}}^* M W_{\text{nopt}}} \quad (29)$$

to obtain a plot of the improvement factor  $\beta_n(\omega)$  of the  $n$ th filter versus an arbitrary target doppler frequency  $\omega$ . The numerator of (29) would be the frequency response of the filter if  $S(\omega)$  was not weighted by the antenna pattern. When  $\omega = \omega_n$  eq. (29) reduces to eq. (25) and represents a point of tangency between eqs. (29) and (25). Figures 7 and 8 are plots of eq. (25) and (29). Although they show the best possible MTI improvement, the side lobes are not adequate for heavy rain rejection.

#### B. Suboptimum Filters

In the Introduction it was pointed out that one should try to approach the optimum improvement (Eq. 25) while at the same time have low side lobes to reject rain. In addition the algorithm should be relatively simple in order to save on processing hardware. There is no one algorithm that is best for all clutter covariances (spectra) and side lobe levels. A number of algorithms were tried, using the AROH parameters. The "best" of these consists of a three-pulse canceller followed by a post-weighted FFT. A post-weighted FFT,  $P_k$ , is defined here as the subtraction of a fraction of the output of two adjacent filters from the filter of interest. In vector notation

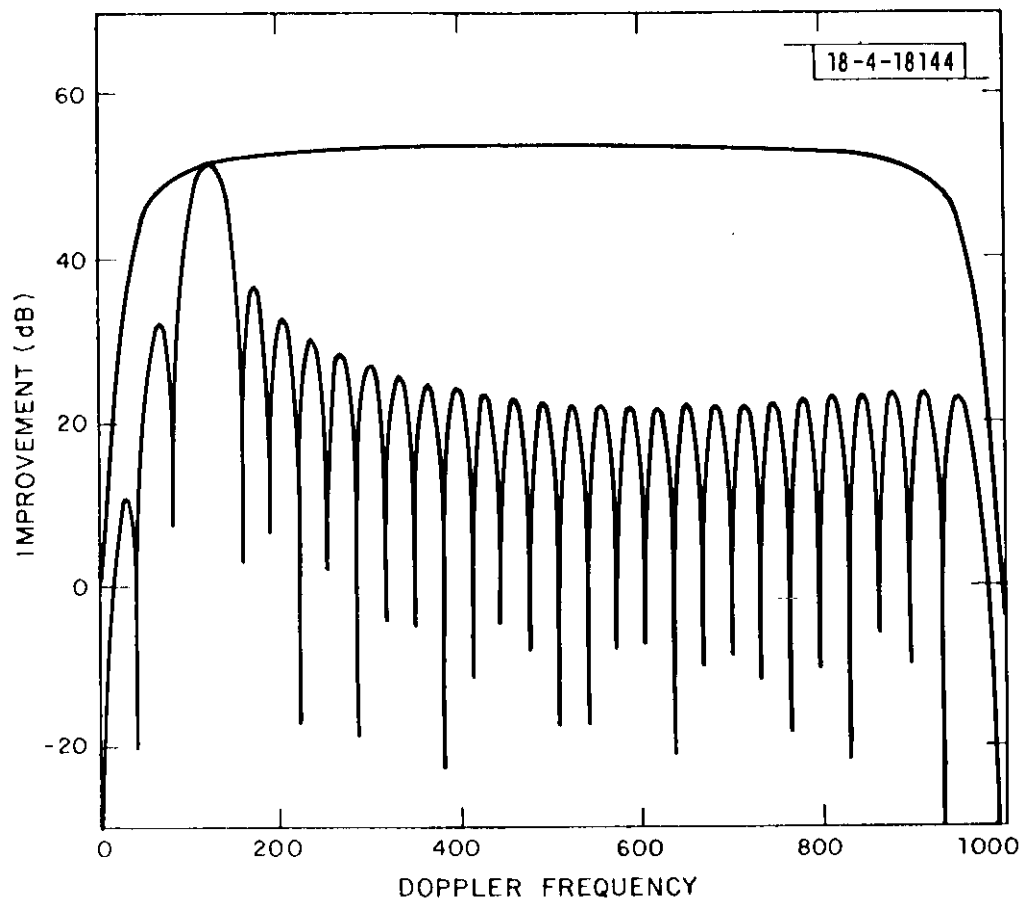


Fig. 7. Optimum Processor Performance for the Fourth (K-4) Filter.

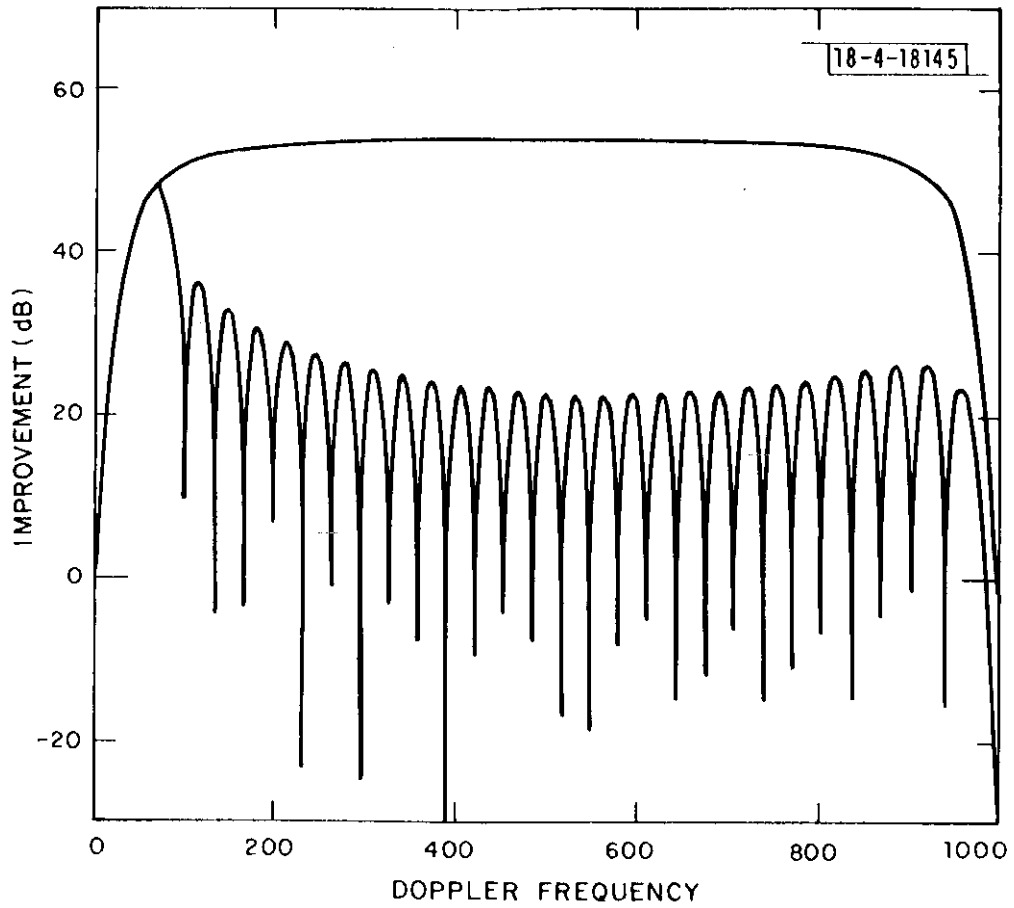


Fig. 8. Optimum Processor Performance for the Second (K-2) Filter.



$$P_k = F_k - \alpha (F_{k+1} + F_{k-1})$$

where  $\alpha$  is a simple real fraction,  $k$  is the filter number and  $F_k$  is the vector notation of an unweighted FFT.  $F_k$  is given by

$$F_k^* = 1, e^{-i \frac{2\pi k}{N}}, e^{-i \frac{4\pi k}{N}}, \dots, e^{-j 2\pi \frac{(N-1)k}{N}} \quad (30)$$

The output of an unweighted FFT is

$$F_k^* S = \sum_{n=0}^{N-1} e^{-i \frac{2\pi nk}{N}} S_{n+1} \quad (31)$$

and the post-weighted FFT is

$$\begin{aligned} P_k^* S &= \sum_{n=0}^{N-1} e^{-i \frac{2\pi nK}{N}} S_{n+1} - \alpha \sum_{n=0}^{N-1} e^{-i \frac{2\pi n(k+1)}{N}} S_{n+1} - \alpha \sum_{n=0}^{N-1} e^{-i \frac{2\pi n(k-1)}{N}} S_{n+1} \\ &= \sum_{n=0}^{N-1} (1 - 2\alpha \cos \frac{2\pi n}{N}) e^{-i \frac{2\pi nK}{N}} S_{n+1} \end{aligned} \quad (32)$$

Thus it is seen that this form of post-weighting is the same as amplitude weighting the input of the FFT. This type of weighting (enclosed in the parentheses) is known as "cosine square on a pedestal" and is commonly used to obtain low sides in filters and antennas. When  $\alpha$  equals .426 it is known as Hamming weighting. As used here, post-weighting consumes less processor time than pre-weighting.

The three-pulse canceller is represented by the following  $(N+2)$  by  $N$  matrix



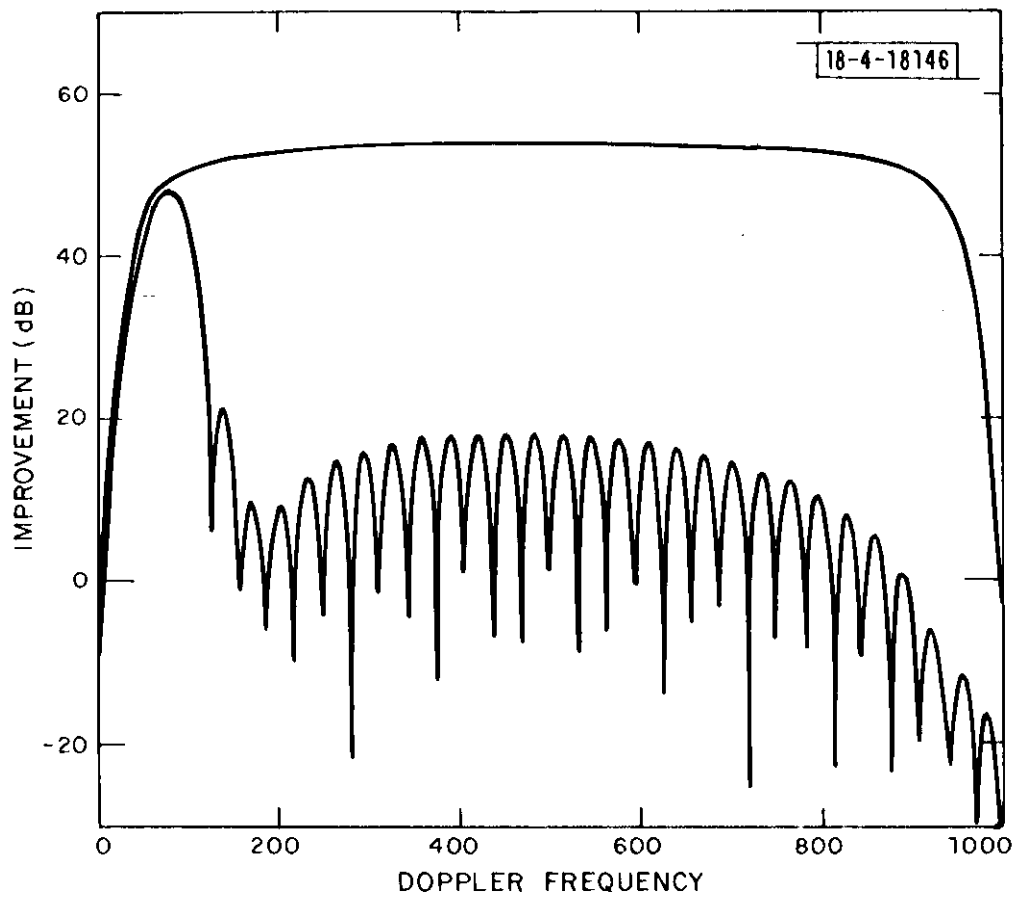


Fig. 9. Suboptimum Processor Performance - K-2 Filter.

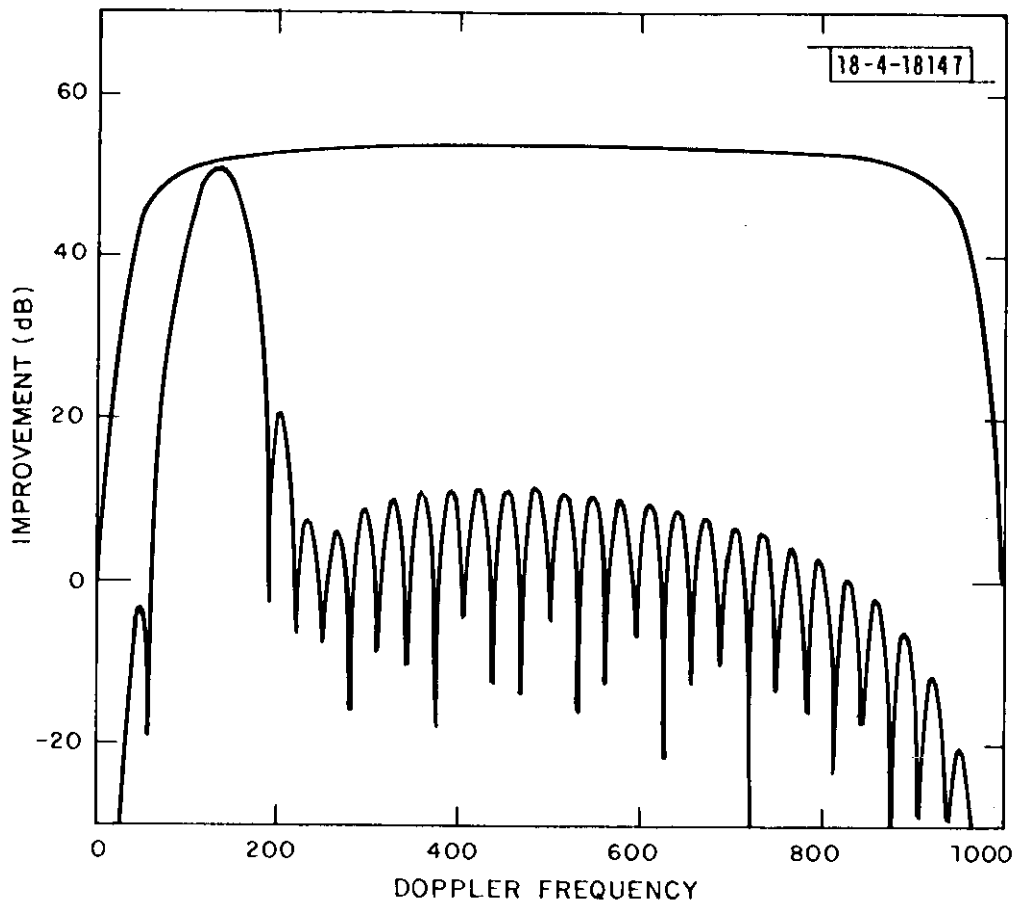


Fig. 10. AROH Suboptimum Performance K-4.

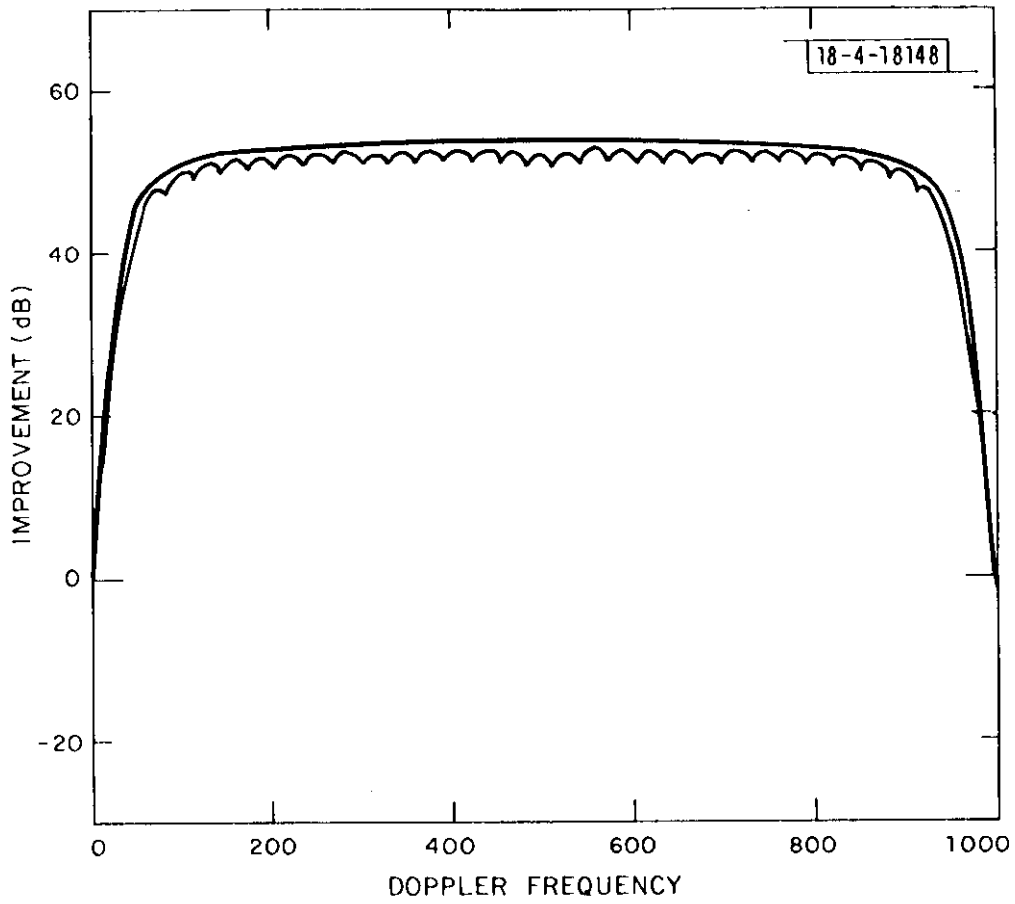


Fig. 11. Composite of Suboptimum AROH Filters.

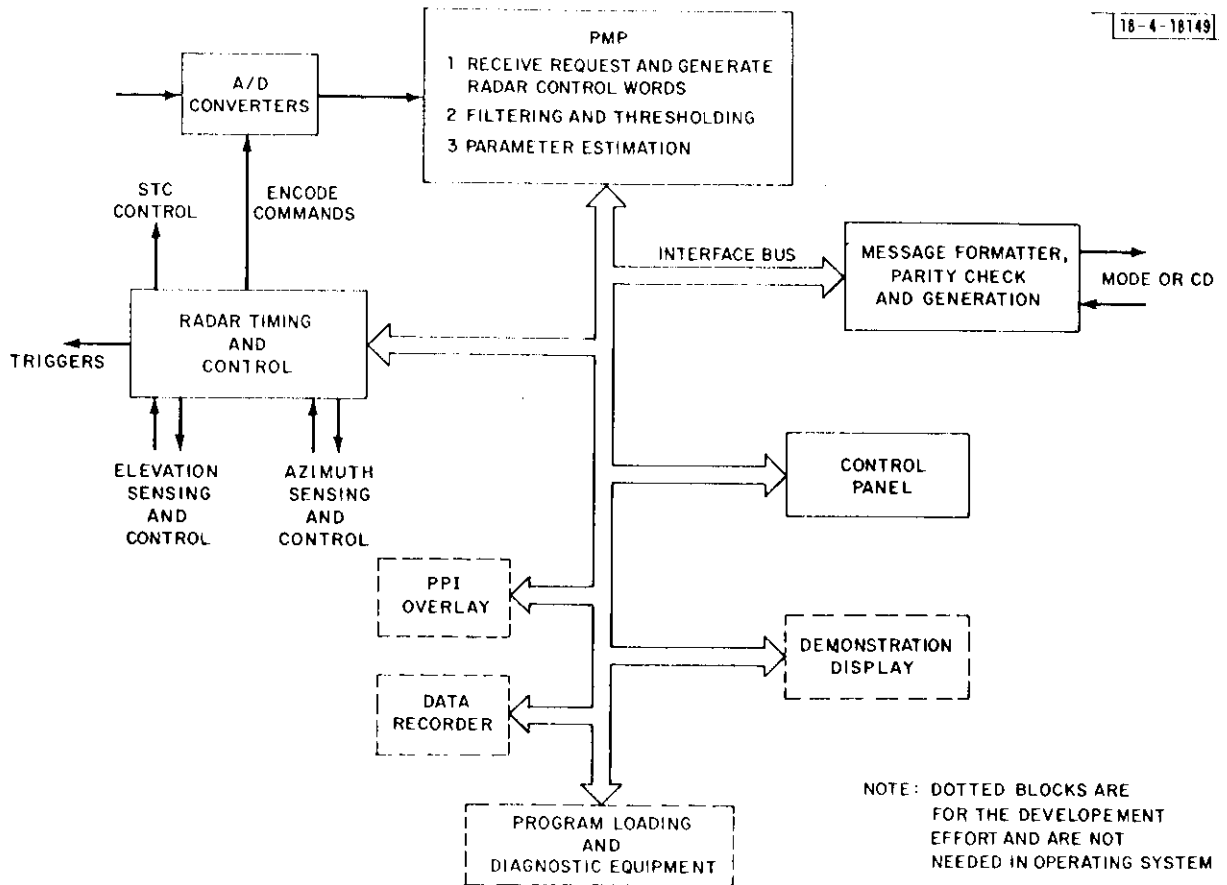


Fig. 12. AROH Digital System.

control system a central processing unit (CPU) performs all the control and computation functions with control signals passed between subsystems and the CPU over a multiplicity of dedicated cables. The CPU and its control programs are needed for testing of the subsystems in these older systems. Similarly, if a subsystem is modified both the controls and the program may need to be changed.

One can imagine a more decentralized system. In such a system each subsystem will have enough computational power and storage to be essentially a stand-alone unit. With the advent of relatively low cost microprocessor integrated circuit chips such a decentralized system becomes economically feasible as well as architecturally attractive. Thus, in the AROH system local control functions will be performed by microprocessors within the subsystems. Only commands and replies or data need to be exchanged between the different subsystems. This decentralized system will obviate the necessity for the CPU to be in operation in order to test the individual subsystems. In general, this decentralized architecture makes partitioning and parallel development of the various subsystems possible. For example, operation and testing of the radar and the radar control functions can be carried on without the need for the big central signal and data processor to be on line. Similarly, the CPU and its programming will be simpler since it can concentrate on signal and data processing without the burden of a complicated interrupt structure for performing the control tasks.

The common bus chosen for the AROH is the new IEEE standard 488-1975 Interface Bus (IB). It is a bit parallel, byte serial type of information exchange system. It has a 16-bit wide path of which 8 bits are used for data bytes. The remaining 8 bits are used for bus control functions. The maximum data rate is nearly 500,000 bytes per second, which is ample for the AROH application. The IB may be viewed as a party line where a number of parties may engage in conversations (i.e., information exchanges) on a time-shared basis. These conversations may involve more than two parties, however only one source (talker) is allowed to output data to the bus where multiple receivers (listeners) may accept the data simultaneously. The time sharing of the bus is coordinated by a bus controller. The application of the IB can be very flexible. The following scenario may explain its operation.

In the AROH digital control system the height request will be processed by the FYQ-47 (CD) interface. This CD interface will convert the request into the range, azimuth, old height and tasks words in the IB format. These will be sent to the PMP signal processor, radar control and local control panel displays. This distribution can be accomplished in a single bus transaction. The request information will be used by the PMP in the target correlation and interpolation process. It will be displayed to site personnel for monitoring. It will be used by the radar controller to compute the search limits and radar PRF.

The radar controller will then initiate the scanning process. The range gate data will be fed by the A/D to the PMP directly for processing. During the scanning process the antenna positions will be reported by the radar controller to the PMP and control panel periodically. At the end of the scanning the radar controller will inform the PMP so that the PMP may complete the correlation/interpolation process and generate the target report. The target report will be sent to the FYQ-47 interface for reply processing and to the control panel for display. Figure 13 shows the sequence of transactions on the interface bus.

#### A. Azimuth Control

The azimuth position control consists of an azimuth position counter and index control circuits. The output of the azimuth position counter will be compared with the output of an azimuth position encoder. The difference will be used by the servo system for driving the azimuth servo motor. In the scanning process the azimuth control receives the start and stop positions from the microprocessor. In addition it will receive a start command and an increment control word.

The increment control word is needed so that the azimuth motion will have a fixed relationship to the elevation nodding motion. This is to optimize the scanning process. The relation desired is for a complete elevation nod to cover the azimuth beamwidth. Basically the increment control word value is accumulated in an azimuth increment accumulator. When this accumulator overflows the azimuth counter reaches the stop position, a comparator output will stop the scanning process and produces an "end of scan" signal. For test purposes, any fixed



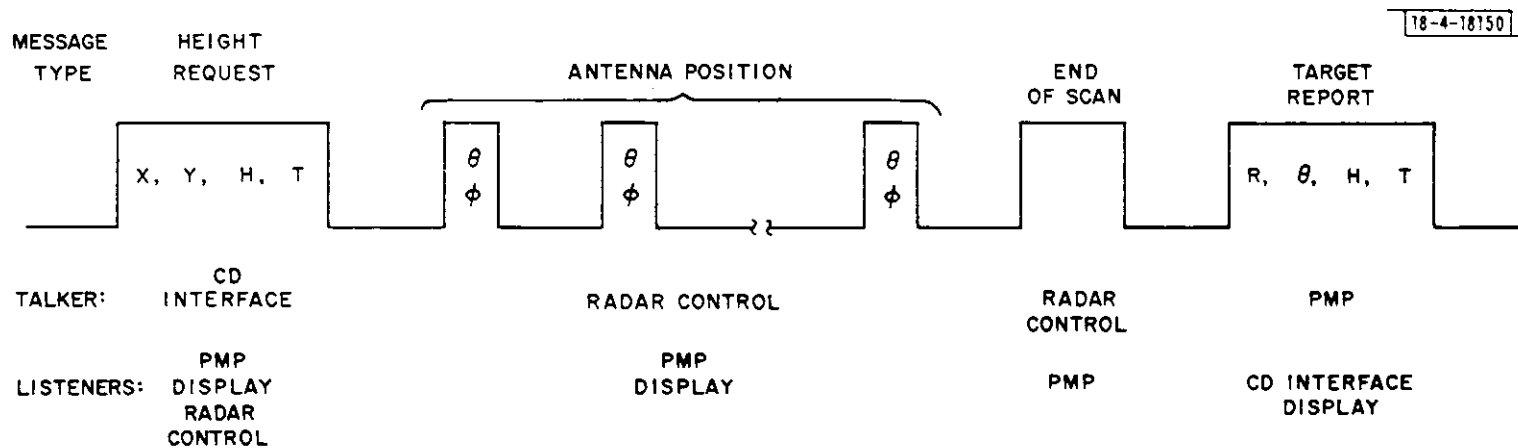


Fig. 13. Interface Bus Transactions.

azimuth position can be achieved by merely loading the azimuth counter without giving the start command.

#### B. Elevation Control

The antenna elevation control receives the maximum and minimum elevations from the microprocessor in the radar controller. These will be stored in the up and down registers of the elevation control. The content of an up and down elevation position counter will be compared with these two limits. The results of the comparison will be used to gate the up or down counting process. For example, if the elevation position counter is greater than the up register, then the up comparators  $\geq$  output will be true. On the next elevation index pulse the up-down FF will be set to the down state reversing the counting process.

The output of the elevation position counter is constantly compared with the elevation shaft position encoder. The difference is the error signal and is converted into an analog signal for use by the servo system. For testing purposes, it will be desirable to fix the antenna at a selected elevation. To achieve that one merely loads the desired elevation to both the up and down registers. The elevation control will automatically step to the wanted position and stop.

For the AROH radar signal processing, a fixed number (48) of pulses will be used to cover a fixed increment ( $0.75^{\circ}$ ) in elevation. Thus, the elevation change rate must be a function of the PRF. In order to achieve this relation a scaling of PRF is needed to arrive at the correct indexing rate. A simple adder, accumulating the proportional factor will be used to do the radar conversion.

#### C. Radar Timing Control

The radar timing control receives the pulse repetition period (PRP) value ( $1/\text{PRF}$ ) from the microprocessor. Two such values are given for each scanning process, one to be used for the down portion of the nod and another, at 20% slower rate, to be used for the up portion. These two values will be loaded into two PRP registers. Depending on whether the antenna is in the up or down nod cycle, the corresponding PRP register will be used to control the pulse repetition period. The operation of the range (pulse period) counter is straightforward. The counter always counts up. When it reaches the specified count (time) the comparator

generates a coincidence signal which will reset the counter to a predetermined negative number. This is the pretrigger period. The counter will count up through zero to the predetermined PRP and repeat the process again. The pretrigger period will be selected so that zero count equals zero range. The counting clock will be derived from the radar IF to minimize interference. The counting clock will also be used to control the A/D sampling process and its period will be chosen to be equivalent to 1/16 of a nautical mile.

#### D. Control Monitor Panel

Since the AROH system is completely automatic, machine interactions at the radar site will be minimal. A control panel is provided for monitoring the system operation, manual inputting of local meteorological data and operation of test and diagnostic subroutines. In normal operation this panel will have alphanumeric displays for the last height reply, the latest height request and the instantaneous position of the antenna and range gates. In addition, a keyboard is provided for inputting meteorological constants and initiating test routines. A "maintenance" switch allows site personnel to take control of the system when that is necessary. A sketch of a preliminary design for that panel is presented in Figure 14.

#### E. Operating Modes

Normal operation of the system will be, as its name implies, fully automatic. Height requests will come from the direction center and be serviced completely automatically. (Panels like that shown in Figure 14 will be available so that site personnel can monitor the operation of the system.) Development and initial testing of the system, however, will be done at Lincoln Laboratory and will be independent of any direction center. For the shakedown and initial testing of the system various degrees of local control will be required. The decentralized nature of the system makes the implementation of such controls relatively easy. All that is required is to arrange for suitable words to be put onto the interface bus. The different control modes that are to be implemented in the development phase are discussed briefly below.

##### 1. Searchlight

The system will be arranged so that the keyboard on the control panel can be used to input a set of coordinates. In this mode the antenna will be slewed to

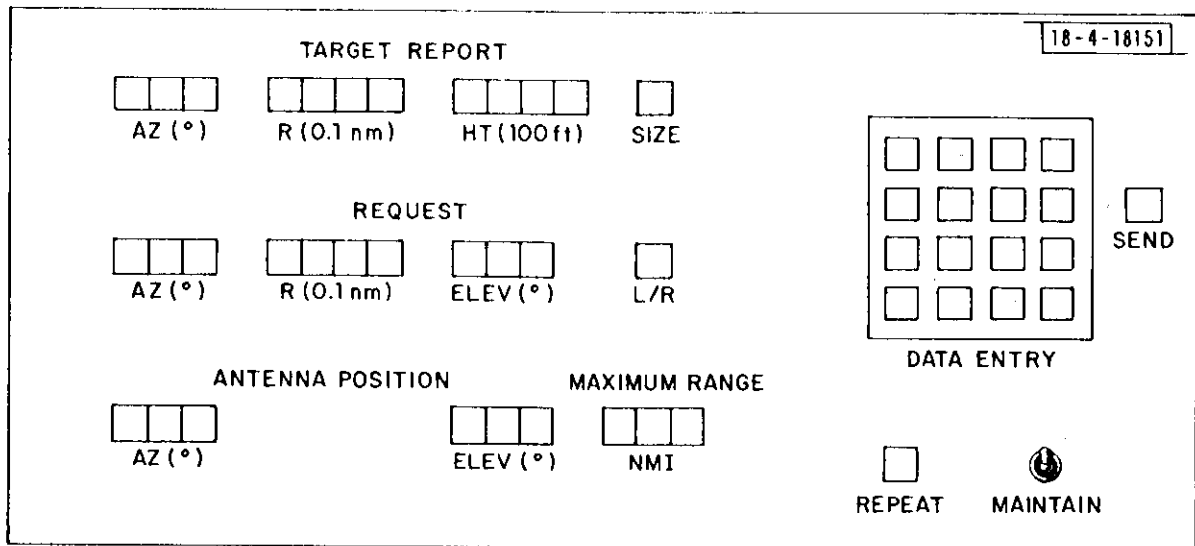


Fig. 14. Radar Control Panel.

the inputted azimuth and elevation and the block of range gates to be processed will be moved to the inputted range. It is expected that this mode will be useful for checking servo responses and accuracy under various environmental conditions and, in conjunction with special processor programming, for measuring the characteristics of the radar environment.

## 2. Keyboard Height Requests

The keyboard can be used to input a complete height request, simulating a request from a direction center. Once such a request is inputted the system will go through the entire response cycle of scanning and outputting any measured target heights. The primary purpose of this mode is to check out the operation of the system. In an operating system, however, it could conceivably be used for local operation with the height outputs either transmitted automatically or manually to the direction center.

## 3. Local Analog Inputs

For development it is planned to use the system with a standard FPS-6 PPI overlay mounted on a PPI. This arrangement will be used to designate targets in polar coordinates as part of the overall testing of the system. A similar arrangement can be made available for SARAH operation during testing at an operating site.

## IX. ELEVATION SCANNING

The AROH system requires an elevation scan which is different from that of the FPS-6. The latter uses a geared AC motor, a crank and a connecting rod to produce an essentially sinusoidal scan in elevation. It nods from -2 degrees up to +32 degrees and back 20 times a minute. At a constant altitude the target elevation angles decrease as the range increases. Hence the FPS-6 spends only a very small portion of its time illuminating the longer range targets. A much greater fraction of the time is spent with the antenna pointed at relatively high elevations where targets of interest are only present at the shorter ranges. This situation is presented in Figure 15.

The signal and data processing in AROH are designed to operate with a fixed number of pulses per elevation beamwidth. As noted earlier, the PRF is set as a function of target range. Two PRF's are specified at each range in order that blind speed targets may be dealt with properly. The higher of the two allows 40

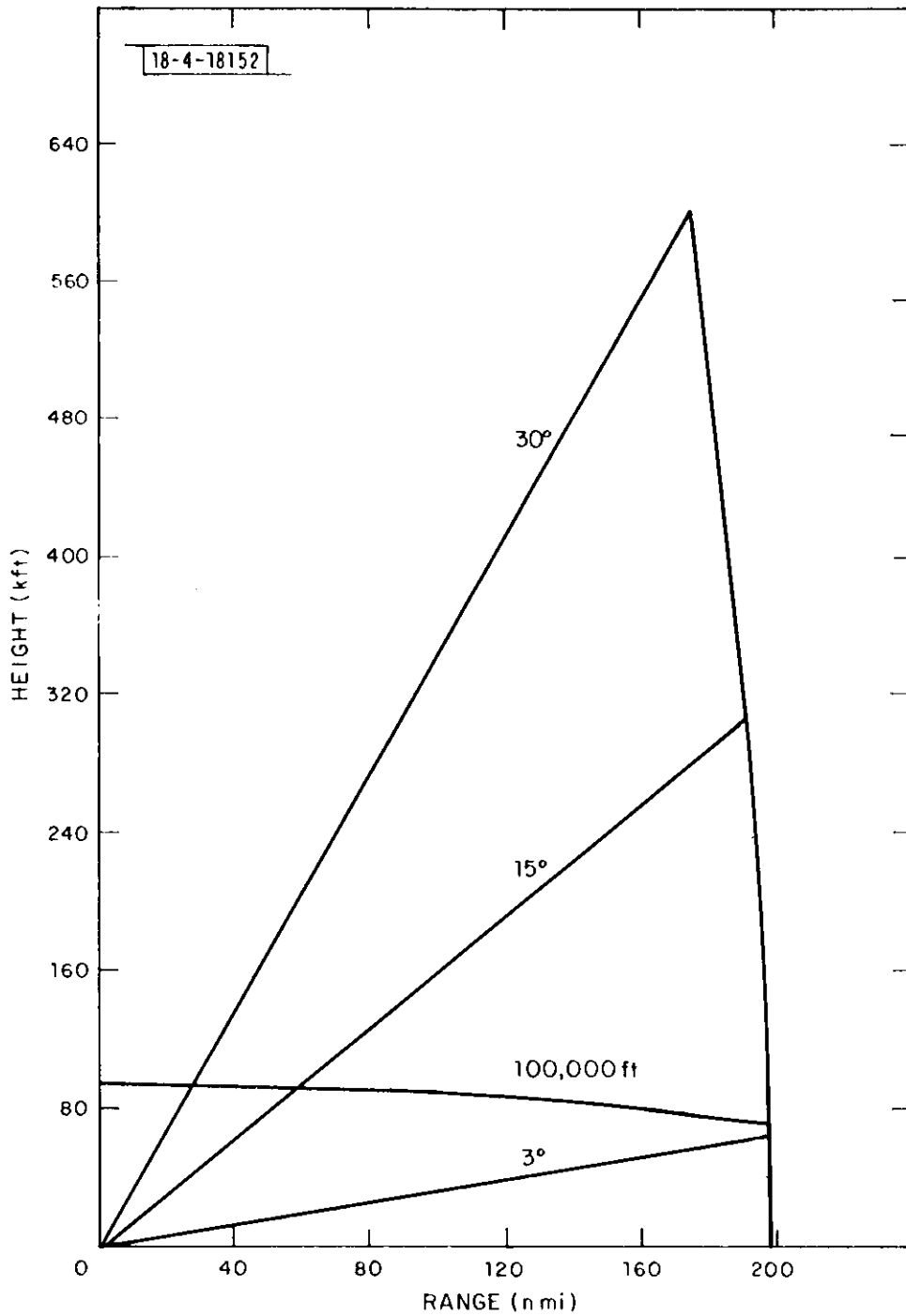


Fig. 15. FPS-6 Elevation Coverage.

microseconds of dead time after the echoes from targets at the longest range. The lower PRF is set at 80% of the higher one at each range.

A linear vertical scan is used. The antenna vertical scanning rate is adjusted as a function of the PRF such that 48 pulses are transmitted while the antenna moves through 0.75 degree in elevation (see Figure 16). The amplitude of the vertical scan is adjusted to cover an altitude range of 100,000 ft at the range of interest unless an estimate of the target height is made available to the system. The maximum elevation scan angle required is plotted as a function of target range in Figure 17.

The higher of the two PRF's is used on the descending scan and the lower when the elevation angle is increasing. For this preliminary design, it was assumed that the maximum acceleration in elevation will be approximately  $37^\circ/\text{sec}^2$ . This value is about half of the maximum acceleration experienced during the 20-nods-per-second sinusoidal scan now being used.

With given maximum and minimum elevation excursions, elevation scan rates and acceleration; the time (in seconds) for a complete elevation nod cycle is given by

$$t = 2.25 \frac{(\phi_{\max} - \phi_{\min})}{\dot{\phi}} + 3.6 \frac{\ddot{\phi}}{\dot{\phi}} \quad (36)$$

where  $\phi$  is the elevation angle

$\dot{\phi}$  is the elevation rate at the higher PRF

$\ddot{\phi}$  is the acceleration

(it is assumed here that acceleration vs time function consists of rectangular pulses applied at the top and bottom of the nod).

The period of a complete vertical scanning cycle is plotted as a function of range in Figure 18.

Obviously to achieve all this variation in elevation scanning, the simple motor drive of the FPS-6 must be replaced with a servo system.

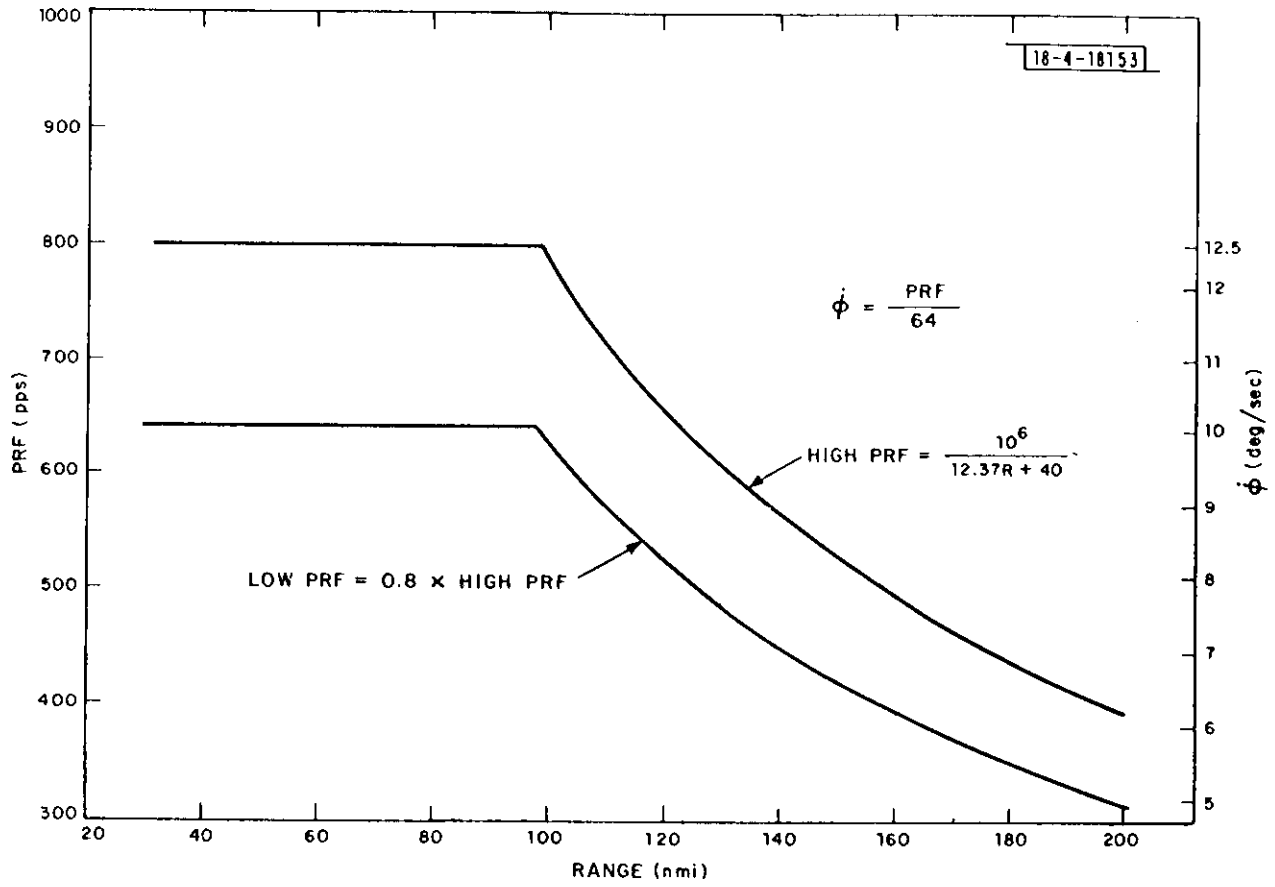


Fig. 16. FPS-6 PRF and Elevation Scan Rates.



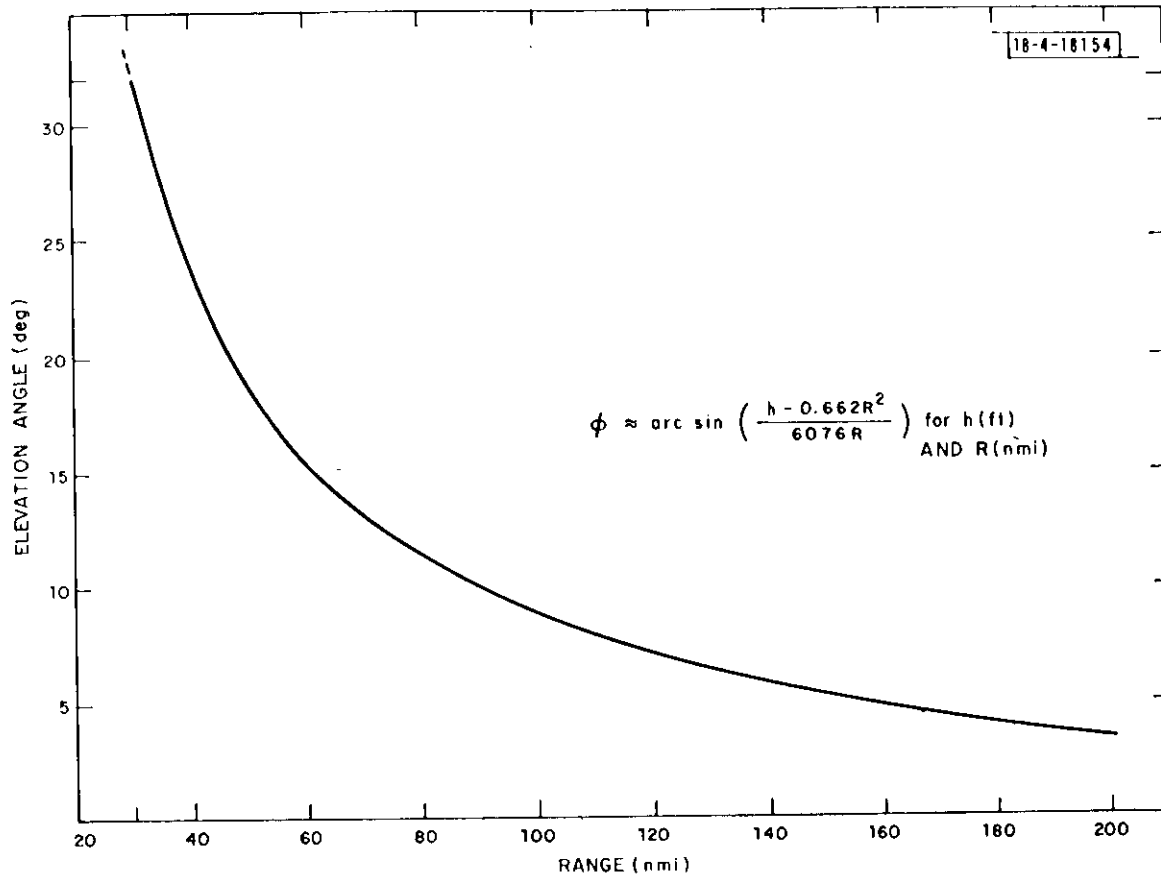


Fig. 17. AROH Maximum Elevation Angle Required for 100,000-ft Coverage.

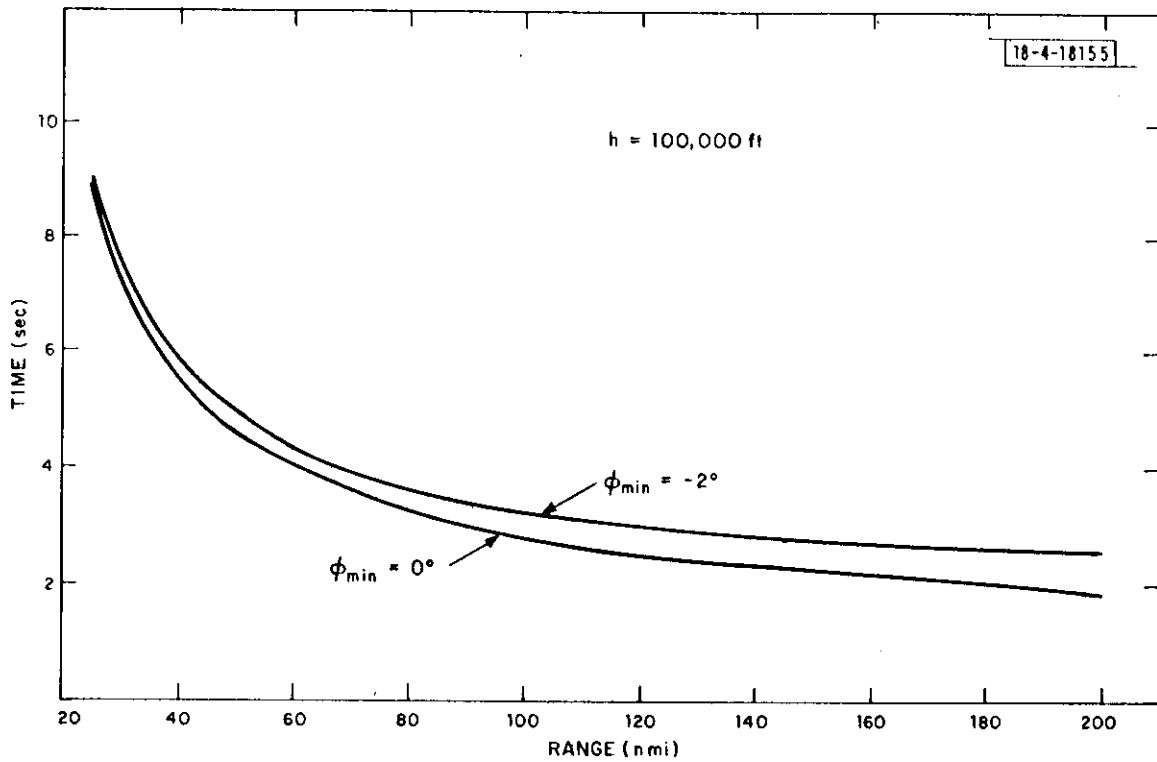


Fig. 18. AROH Time for a Complete Nod.

Although one version of the FPS-6 has been equipped with a hydraulic elevation servo mechanism, it was decided not to use the hydraulic approach for the following reasons. First, the hydraulic system "power supplies", i.e., pumps, accumulators, etc., have to be carried on the elevation yoke and put an additional inertial load on the azimuth servo system. Second, hydraulic systems tend to leak fluid and become messy and dirty. Third, hydraulic systems are inefficient energy users. Finally, the electronic technicians who will be responsible for maintenance are, in general, not familiar with hydraulic systems.

It has been decided to use a technique that has seen heavy service in industrial numerically controlled (NC) machine tools. Specifically, the drive will consist of permanent magnet servo motor that drives a ballbearing lead screw. The nut on the lead screw will be connected to a push rod that will attach to the back of the antenna dish. The assembly will be enclosed in a weatherproof, dust-free box with the push rod coming out of one end of the box. The opposite end of the box will be attached to an arm extending out from the back side of the yoke (see Figure 19).

We expect the servo system to have the following characteristics. The elevation readout will be effected by a 14-bit optical encoder that is directly coupled without the use of gears. It will be mounted in place of the old elevation synchro. The encoder will resolve to 1.3 minutes accuracy or to 1/40 of a beam-width. The digital output of the encoder will be compared to the computer command then converted to analog to drive the servo motor amplifier. The servo amplifier is to be of the pulsewidth modulator (PWM) type. This will, in turn, drive the PM motor. The PM motor will have a 3-HP continuous rating. The motor is to drive the ballbearing lead screw having a pitch of .5" per revolution. The servo system will have a wide bandwidth in order to handle sudden wind gusts. Wind loadings of greater than 1,000 lbs. should not have any appreciable effect on pointing accuracy.

The elevation drive is purposely over-designed to minimize failures. An oversized (in diameter) lead screw should be failure free for many years. The motor is also oversized and, consequently, its brushes may also last for years before replacement. The motor will contain an integral brake which is automatically

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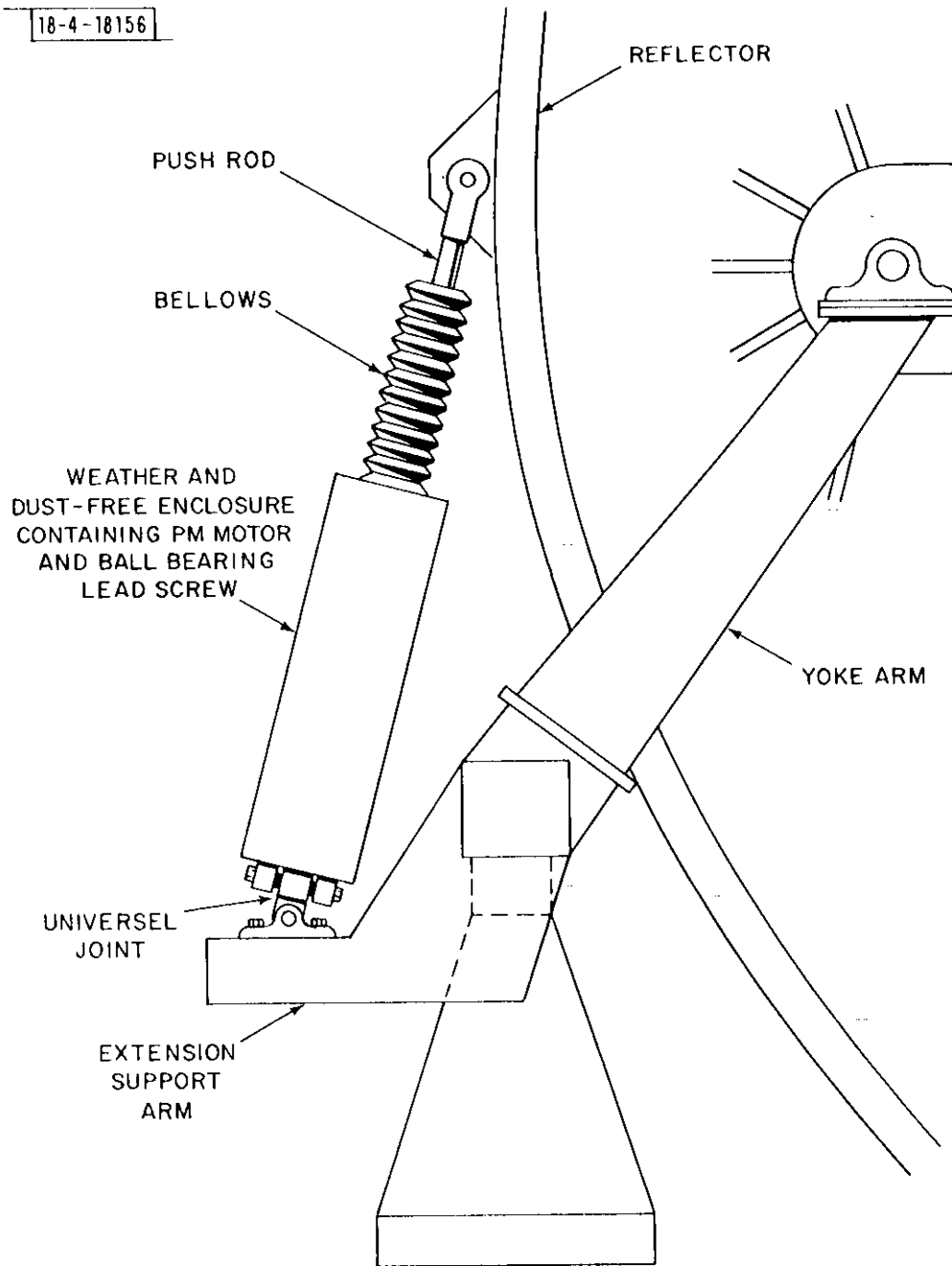


Fig. 19 . Elevation Servo Drive .

applied if there should be a power failure or an override of the travel to limit switches.

#### X. AZIMUTH SEARCH

In normal operation of the SAGE system there is often an error of as much as a few miles between the position coordinates in a height request and the true position of the desired target. The SAGE semiautomatic height finder operator has an azimuth control which he can use to steer the antenna in azimuth if he does not see a target at the requested position. This azimuth control function is derived from the common digitizer which limits the travel in azimuth to +5 miles from the azimuth of the height request at the range of interest. It is planned to program the AROH to search in azimuth to the same limits. Thus, when it receives a height request the AROH system will scan in azimuth from 5 miles on one side of the requested target position to 5 miles on the other side. The actual azimuth excursion involved varies inversely with the target range from +1.43 degrees at 200 nmi range to +9.6 degrees at 30 miles.

In the interest of assuring proper detection we require that the antenna go through a complete up and down nod at each position covered. Hence, the total number of nods devoted to searching for a target is always in integer. The azimuth scan rate is adjusted so that the antenna scans one azimuth beamwidth for each complete elevation nod. The number of nods required varies from unity at the longer ranges to six at the very short ranges. The time required for completion of a nod increases as the range decreases (Figure 17). This effect combined with the increasing number of nods required at the shorter ranges to make the overall time required for the search to vary from a few seconds at the longer ranges to 30 or more seconds at the very short ranges. This total search time is plotted as a function of range in Figure 20.

The time required for a complete search over a 10-mi by 10-mi by 100,000 ft volume gets quite long at the shorter ranges (30 seconds at a range of about 30 miles). Thus a requirement for a 15-sec response time is not compatible with optimum performance over such a large search volume at the shorter ranges. Hopefully in actual operation such a large search volume will be needed seldom, if at

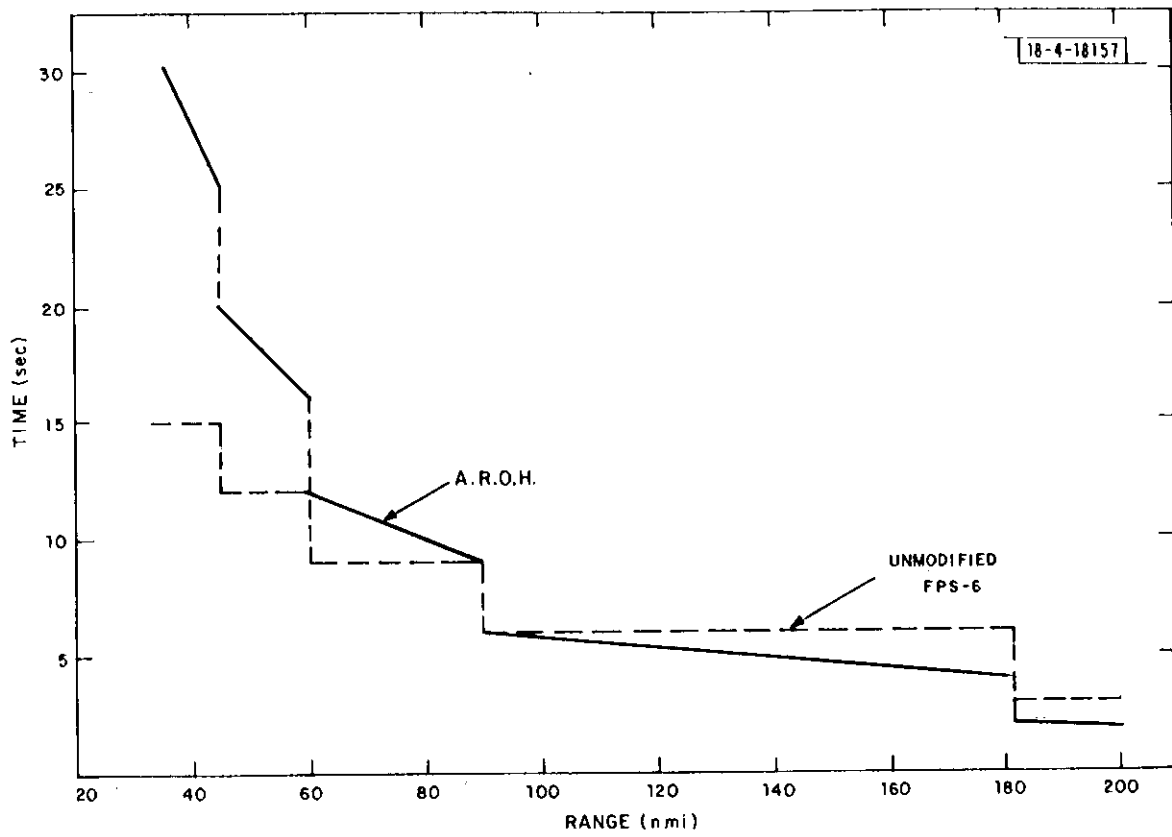


Fig. 20. Approximate Times to Search 10-mi by 100,000-ft Sections.

all. For example, if prior or expected height information is available the altitude dimension of the search volume can be reduced significantly. Thus, if the target were known to be below 50,000 ft the search time could be cut almost in half. Similarly if the maximum altitude could be reduced to 70,000 ft and the cross range dimension could be reduced from 5 miles to 3.5 miles, the search time would be reduced to about half of the value given in Figure 19. In any case, some sort of compromise between the conflicting requirements of large target search volume, short range, fast response time and good performance must be negotiated. This area (that of short range response time) appears to be a minor one and the only one over which such negotiation will be required.

It is hoped to use the existing FPS-6 amplidyne azimuth drive for AROH. Accordingly, digital azimuth output from the AROH system will be passed through a digital-to-synchro converter for input to the FPS-6 azimuth control system. Similarly, the azimuth synchros in the FPS-6 will be retained and their signals will be passed through synchro-to-digital converters for use in the AROH digital system.

#### XI. STABILITY MODIFICATIONS

To support near optimal digital signal processing, the analog portion of the radar must have a large dynamic range and it must be free of spurious modulations which produce sidebands in the doppler bandwidth occupied by expected targets. Radars which, like the FPS-6, were designed in the early 1950's were simply never intended to produce this level of performance. Thus, part of the process of adding high performance digital processors to existing radars is modification of the radars themselves. These modifications include providing receiver channels with increased linear dynamic range as well as cleaning up the spurious modulations in the system. During the development of the MTD at Lincoln Laboratory this process of modifying the analog portion of the radar has become known as "stability modifications". In general it consists of providing a receiver channel with adequate dynamic range and linearity and then identifying and eliminating all the causes of unacceptable spurious modulations. Important modifications which are known to be required for the AROH program are discussed briefly in the following sections.

A. Stalo

The stable local oscillator which operates within a few tens of a MHz of the radar frequency is usually the most important limitation on the radar's ability to reject clutter. The FPS-6 is not an exception. It uses a reflex klystron stalo which, while suitable for its original purpose, has much too much low frequency spurious angle modulation to be useful in an AROH system. This stalo will be replaced with a crystal-stabilized, solid-state oscillator. This latter will be of a type which has been used extensively in various MTD radar projects.

B. Transmitter

When the FPS-6 was introduced in the early 1950's it was an extremely high powered radar. In order to get this high power from a single magnetron with the technology of that time, some compromises were made. In the magnetron a relatively high level of spurious output was tolerated in order to achieve the very high power level. This has come to be unacceptable in today's environment of emphasis on electronic compatibility between equipments. Accordingly, the Air Force is implementing a modification to the FPS-6 which involves replacing the older magnetron with a new coaxial magnetron design. This coaxial magnetron appears to be able to provide MTD level stability. However, some changes in the modulator system will undoubtedly be required.

In the first place, as noted previously, to keep the system within its average power limitations while operating at the higher pulse repetition rates required for AROH, the pulse width must be reduced to 1 microsecond. Unfortunately, in order to operate without producing energy from unwanted modes, the coaxial magnetron requires a relatively slow rate of rise during the top of the leading edge of the video pulse. In the present modification this is being obtained by adding inductance external to the pulse forming network. In addition to slowing down the rise time of the video pulse, this inductance also slows the pulse decay time significantly. Ideally the coaxial magnetron would get a relatively slow rate of rise, particularly over the top 10 percent or so of the video voltage and a fast decay at the end of the pulse. It is felt that the presently used linear modification may lead to an unacceptable spectrum when the pulse is shortened to 1 microsecond.



Lincoln Laboratory has used a nonlinear network to overcome this problem in testing a different coaxial magnetron. The network used diodes, a small capacitor and some Zener diodes. It had the effect of reducing the rate of rise during the top 10 percent of the leading edge but did not affect the trailing edge of the trailing edge of the video pulse. This was done on an 0.9-microsecond pulse and resulted in a clean symmetrical spectrum from the coaxial magnetron. That coaxial magnetron ran at a power level somewhat less than a megawatt. A similar implementation of a similar network in the FPS-6 appears to be the best approach to getting good operation at the shorter pulse width.

Other changes in the transmitter will include the addition of a holdoff diode to allow stable operation over the 2-1 variation of pulse repetition rate and the design and installation of a new pulse forming network for the shorter pulse width.

#### C. Coho

AROH, being a coherent system, requires a coherent reference. The FPS-6, being a noncoherent system, has none. Hence, a part of the AROH development will be the provision of a coherent oscillator for the modified FPS-6. It is anticipated that an existing design will be adequate with minor modification.

#### D. Receiver

As in all previous applications of MTD to existing radars, the AROH will be provided with a completely new channel. This will include a solid-state mixer, solid-state intermediate frequency preamplifier and amplifiers, a passive intermediate frequency bandpass filter and solid-state quadrature video detectors with video amplifiers. This modification is required because the older receiver was designed to provide only 15 dB or so of dynamic range. The AROH system will require more than 50 if the full capability of the analog-to-digital converters is to be realized. In order to utilize the receiver dynamic range most efficiently a digitally controlled sensitivity time control (STC) will be implemented.

## REFERENCES

1. P. M. Woodward, Probability and Information Theory with Applications to Radar (McGraw-Hill, New York, 1953).
2. G. W. Deley, "Waveform Design," in Radar Handbook, edited by M. Skolnik (McGraw-Hill, New York, 1970), pp. 3-1 - 3-47.
3. M. I. Skolnik, Introduction to Radar Systems (McGraw-Hill, New York, 1962), p. 113.
4. D. K. Barton, Radar System Analysis (Prentice Hall, New York, 1964), p. 283.
5. P. Swerling, Proc. IRE 44, 1146 (1956).
6. D. K. Barton, Radar System Analysis (Prentice Hall, New York, 1964), pp. 51-54.
7. L. V. Blake, "Prediction of Radar Range," in Radar Handbook, edited by M. Skolnik (McGraw-Hill, New York, 1970), pp. 2-51 - 2-54.
8. F. Nathanson, Radar Design Principles (McGraw-Hill, New York, 1969).
9. D. K. Barton, Radar System Analysis (Prentice Hall, New York, 1964), p. 106.
10. M. I. Skolnik, "Introduction to Radar Systems (McGraw-Hill, New York, 1962), p. 542.
11. D. Atlas, as reported by F. Nathanson in Radar Design Principles (McGraw-Hill, New York, 1969), pp. 205-213
12. W. H. Drury, "Improved MTI Radar Signal Processor," Project Report ATC-39, Lincoln Laboratory, M.I.T. (3 April 1975), DDC AD-A010478/6.
13. W. J. McEvoy, "Discrete Clutter Measurements in the Metropolitan Boston Area," MTR-2085, The MITRE Corporation (March 1972).
14. D. F. DeLong, Jr. and E. M. Hofstetter, IEEE Trans. Inform. Theory, IT-13, 454, (1967), DDC AD-667873.
15. R. C. Emerson, "Some Pulse Doppler MTI and AMTI Techniques," Rand Corporation Report 274 (1 March 1954).

