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Propagation of Mode S Beacon Signals on the Airport Surface

Many airports across the United States will soon be equipped with Mode S, a next-generation beacon (or secondary) radar system. One feature of Mode S is that it provides a data link between airborne aircraft and air traffic controllers. If Mode S could be used to communicate with aircraft on the airport surface, the radar system would improve airport safety and efficiency on runways and taxiways. The airport surface, however, is a hostile propagation environment. This article outlines a candidate design for the propagation of Mode-S beacon signals on the airport surface. Data that support the feasibility of Mode S for surveilling runways and taxiways are presented.

Late in the afternoon on 27 March 1977, a KLM Royal Dutch Airlines 747 was preparing for takeoff from Santa Cruz de Tenerife Airport in the Canary Island. Unbeknownst to the KLM pilot, a Pan American World Airways 747 was then taxiing head-on down the same runway. In ground fog that limited airport visibility to about 900 ft, the two jumbo jets collided. During the impact, the nose gear of the KLM aircraft passed over the Pan Am's top, and one of the KLM engines ripped off a portion of the Pan Am upper lounge. The collision, the worst accident in aviation history, killed 582 people.

Preventing such disasters on runways and taxiways is a formidable task, particularly in conditions of high traffic density and poor visibility. To prevent midair collisions, air traffic controllers rely on radio communications, and primary and beacon (or secondary) radars. On the airport surface, however, electronic surveillance is provided solely by primary radar, which is limited in its ability to resolve and identify different aircraft. Furthermore, only a small percentage of airports have primary radars that are dedicated for runways and taxiways.

The secondary radar currently in use at airports will soon be replaced by Mode Select, or Mode S [1], a more accurate system that incorporates an air-to-ground-to-air data link. If Mode-S signals could be propagated successfully on the airport surface, then the data-link capability of the radar system could be used to improve airport safety and efficiency on runways and taxiways. In this article, we present data that support the feasibility of using Mode S on the airport surface.

Our proposed design calls for economical Mode-S ground stations for transmission and reception. The stations are diversely located so that high link reliability and multiple coverage are achieved over a very substantial portion of the airport surface. The multiple coverage provides the basis for surveillance position measurements that use differential-time-of-arrival (DTOA) techniques. DTOA surveillance can operate with just the spontaneously emitted Mode-S signals called squitters so that the technique does not interfere with other Mode-S functions.

Brief History of Radio Aids in Air Traffic Control

The purpose of air traffic control (ATC) is to prevent aircraft collisions, either in the air or on the airport surface. Safe separation between airborne aircraft is accomplished with radar-based displays that contain information on aircraft positions. Using the displays and VHF radio communications, air traffic controllers on the ground issue flight instructions to pilots. Figure 1 illustrates the history of radio aids in ATC.

Current radars for surveilling airborne aircraft evolved from designs developed for military use during World War II. The ATC Radar Beacon

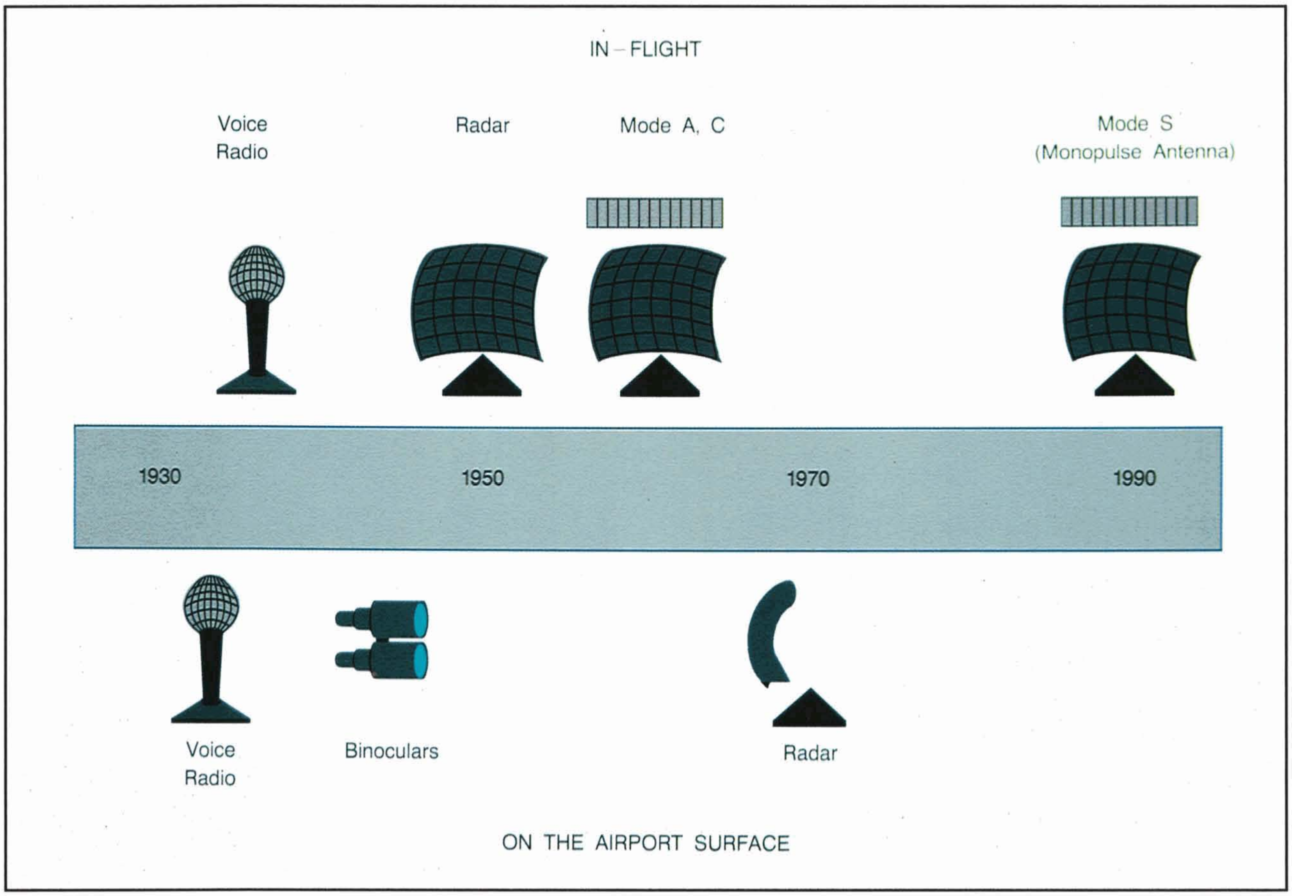

Fig. 1–History of radio aids in air traffic control.

System (ATCRBS) is a secondary, or beacon, radar that grew out of the Identification Friend or Foe system. ATCRBS is a cooperative radar in that it does not rely on the receipt of reflected energy from aircraft. Instead, aircraft carry transponders, which comprise receivers and transmitters. A transponder recognizes interrogations from a radar and transmits a reply. This capability greatly increases the surveillance range of the radar, and enables an identification function when the transponder attaches an identity code to its Mode-A [1] reply. In addition to the identification function, many aircraft connect their altimeters to their transponders so that their Mode-C [1] replies can include altitude information. The ATC system also uses primary, or skin return, radar as a backup to ATCRBS.

Controllers responsible for keeping aircraft from colliding on runways and taxiways do so by visual means, sometimes with binoculars. To augment visual surveillance during periods of low visibility, a few airports have a single primary radar. Unfortunately, these radars do not provide aircraft identities and they do not work well in rain. Furthermore, even though they are mounted on top of the control tower, the radars may not have a clear view of the airport surface; e.g., structures such as hangars and terminal buildings could obstruct the view.

ATCRBS radars used for aircraft in flight cannot surveil the airport surface for two main reasons. The first is that ATCRBS transponders have a turnaround delay between receipt of an interrogation and transmission of the reply. The delay causes a range-measurement uncertainty of several hundred feet, which is tolerable in the air where planes are at least thousands of feet apart, but is unsuitable on the airport surface where as few as tens of feet separate aircraft. The second reason is that every interrogation elicits a reply about 20 μs long from each aircraft that hears it. As a result, the replies from aircraft that are within 1.67 mi of one another will overlap. Since many aircraft are typically on the airport surface, and because the airport dimensions are on the order of a mile or two, virtually all the ATCRBS replies would overlap one another. Consequently, the replies would be rendered totally undecipherable.

During the 1960s ATCRBS began to overload because of increases in the number of aircraft, the percentage of aircraft that were equipped with transponders, and the number of ATCRBS radar installations. The VHF radio circuits also began to strain, and the planned increase in the number of VHF channels was deemed insufficient to handle the traffic densities projected for future decades.

Consequently, the Federal Aviation Administration (FAA) selected Lincoln Laboratory to develop the Discrete Address Beacon System (DABS), a new high-capacity beacon radar that would also include a digital data-link function. To insure compatibility with existing ATCRBS transponders, the Laboratory designed DABS, which has since been renamed Mode Select, to retain all the essential features of ATCRBS [1]. In addition, the Mode-S design includes two major improvements: individually timed and addressed interrogations to Mode-S transponders, and a new antenna that makes monopulse bearing measurements.

The use of individually timed and addressed interrogations solves the synchronous-garble problem because interrogations to aircraft that are close can be separated in time so that the replies do not overlap. Since Mode-S transponders can recognize their own unique 20-bit address, air traffic controllers can use this capability to transmit encoded digital data to specific aircraft. Digital communications from the aircraft to the ground can be appended to the replies.

In monopulse direction finding, the antenna is wide enough so that phase-measuring elements mounted on the antenna face can detect the phase differences that occur when a wave front strikes the antenna at an angle. The phase differences are processed to yield a measure of the incident angle. The measurement, which is accurate to about a tenth of a degree, can be made from only one reply, hence the name monopulse.

The monopulse technique contrasts to that used by ATCRBS. In ATCRBS, interrogations are done so frequently that about 20 to 40 replies are received as a fan beam that is narrow in azimuth and broad in elevation sweeps by a target. The target's bearing is taken to be the antenna's pointing direction at the instant when the middle reply is received.

Since Mode S can operate with only one reply, the rate of interrogations to ATCRBS transponders (i.e., the pulse-repetition frequency, or PRF) can be substantially reduced. For various reasons beyond the scope of this article, Mode S uses a PRF about one-fourth of that used by ATCRBS. This decrease minimizes congestion on the interrogation and reply channels. Because Mode-S transponders are slowly replacing ATCRBS ones, congestion will be reduced even further, since Mode-S transponders require only one reply per antenna rotation.

Air Traffic Safety

Although the ATC safety record is commendable, collisions have occurred over the course of aviation's long history. Table 1 contains excerpts from the official accident reports of the 20 December 1972 Chicago collision and the 27 March 1977 Tenerife tragedy that involved two Boeing 747s. The National Transportation Safety Board's report of the Chicago accident highlights the potential benefit of a surveillance system that provides an identification function as well as sufficient accuracy and coverage, even in conditions of restricted visibility. The Tenerife collision, which killed 582 people, is the worst disaster in aviation history. The Tenerife accident report highlights the potential benefit of data-link communications over voice radio communications, which are susceptible to misunderstandings that arise from noisy transmissions, language differences, and misdirected communications.

Table 1. Official Accident Reports of Two Major Runway Collisions

National Transportation Safety Board's report on Chicago collision (December 1972):

> "[The] probable cause of this accident was the failure of the traffic control system to insure separation of aircraft during a period of restricted visibility."

Dutch report on Tenerife collision (March 1977):

> "The misunderstanding had arisen exclusively from the terminology . . . ; a misunderstanding might also have arisen from such terminologies if there would have been no fog."

> "The unintentional simultaneous transmission of the message . . . generated a squeal, in which both essential messages were lost."

A Candidate Design

It is well known that beacon radars have difficulty surveilling ground-based targets. (The reasons will be discussed in the next section.) In fact, some experts have informally claimed that because Mode-S waveforms are long (tens of microseconds in length) and complex (differential phase-shift keyed and pulse-position modulated [PPM]), it is essentially impossible to propagate the waveforms successfully on the airport surface.

On the other hand, it is reasonable to assume that success could be achieved if the Mode-S radars are very close to the targets. One obvious solution would be to scatter a number of radars around the airport surface so that every target has at least one radar fairly close to it. One drawback of this solution is cost. Position-measuring systems that use both two-way range measurements and direction finding based on beam splitting (as in ATCRBS) or monopulse (as in Mode S) require large, expensive rotating antennas. It is important to note, however, that there is no inherent need for such antennas if the system's only goal is to provide a data link. Thus a design for a data-link-only system could be based on simple and relatively inexpensive omnidirectional antennas.

As stated earlier, two-way range measurements are not accurate enough for use on the airport surface because of the transponder turnaround-time uncertainty. Can the omnidirectional antennas that are proposed for the data-link function be exploited to provide target positions in a way that does not depend on the transponder turnaround time? The answer is yes if three or more antennas receive the replies. In such a case, DTOA solutions based on the intersections of hyperbolas whose foci are the antenna locations will fix the position of the

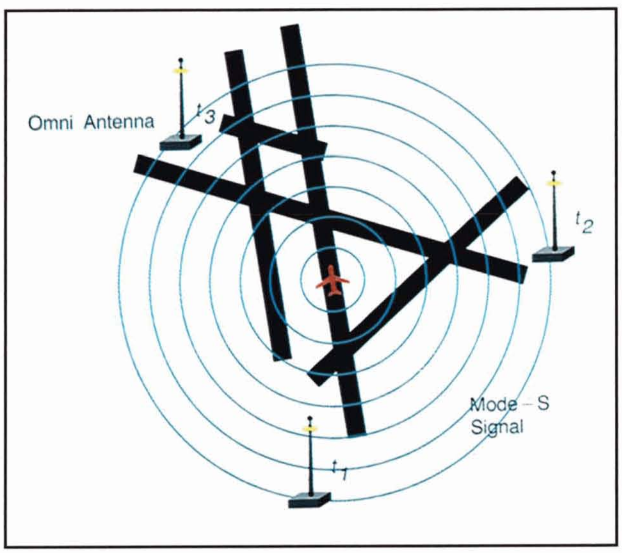

Fig. 2–Candidate design.

emitting source. This method does not require any knowledge of the emission time.

Figure 2 is a sketch of the candidate design. For simplicity, the figure shows only three radar ground stations, all of which might be able to interrogate a target and receive the reply. However, even if only one station can interrogate and receive, a target's position can still be determined if the other two stations can at least mark the arrival time of the reply (Fig. 3). A method for calculating the position for such a case is contained in the appendix, "Position Determination." In this method, the target must be in nearly the same plane as that defined by the three antennas. Except in rare instances, this condition would be met by targets on the ground.

Figure 4 illustrates the entire system for the reply link. The figure shows that the antennas are simple omnidirectional antennas, each equipped with an interrogator and/or receiver. The antennas are all connected by cable to a central processing facility that drives the controller display and/or interface.

Although a quantitative answer would require further investigation, we believe that the proposed surveillance method would not necessarily require significantly more antennas scattered on the airport surface than the number needed to perform the data-link function. However, we note that the data link requires the successful decoding of an entire message waveform on both the interrogation and reply channels from at least one radar station. The reply will provide one time-of-arrival measurement; thus only two more are needed. The antennas that provide the additional two measurements do not require the ability either to interrogate a target or to decode the entire reply message. In fact, the antennas need only be able to recognize the presence of a reply and mark its arrival time.

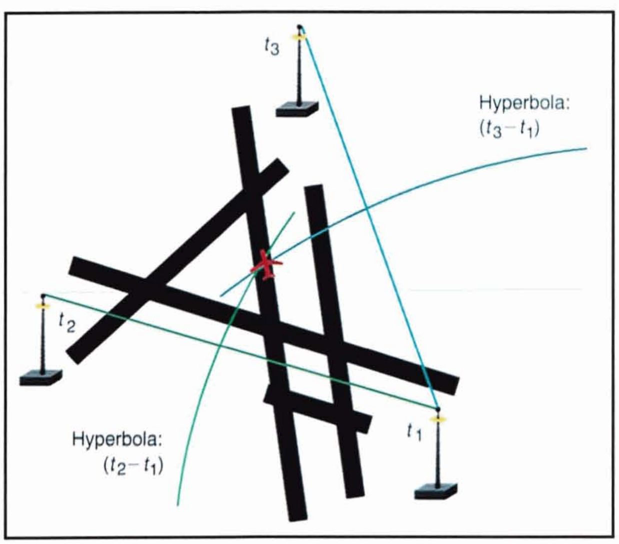

Fig. 3—Position determination using hyperbolas.

Propagation Issues

Propagation problems encountered on the airport surface have three main causes: path-loss attenuation, obstructions, and building reflections.

Path Loss

Path-loss attenuation is the power that is lost between transmitter and receiver. In free-space propagation, a signal with power P_t transmitted from a distance R will result in a transponder receiving a power P_r that is equal to

$$P_r = \frac{P_t}{4\pi R^2}\left(\frac{G_t}{1}\right)\frac{G_r \lambda^2}{4\pi}$$

where G_t and G_r are the gains of the transmitting and receiving antennas respectively, and λ is the wavelength of the transmission. When the transmitting and receiving antennas are respectively h_t and h_r above an infinite, perfectly conducting ground plane, and R is much greater than the sum of those heights, then P_r is approximately given by

$$P_r \cong \frac{P_t}{4\pi R^2}\left(\frac{G_t}{1}\right)\frac{G_r \lambda^2}{4\pi}\,\frac{(2\pi/\lambda)^2(2h_t h_r)^2}{R^2}. \quad (1)$$

Since the ground terrain of airport surfaces are typically close to perfectly reflecting, the propagation losses there nearly follow the above R^4-loss characteristic. Details of the consequences of such propagation losses are contained in the reply-link budget of Table 2. The example assumes a nominal transponder power of 250 W, a minimum trigger level (MTL) of –74 dBm, and an aircraft antenna height of 5 ft above ground level, which is typical for antennas mounted on the bottom of aircraft. If the Mode-S omnidirectional antenna is just 10 ft high, then the margin is only 12 dB when the range is 5,000 ft. If the range is increased to 10,000 ft, then there would be no margin at all to account for such factors as antenna lobing.

The above results indicate the clear need for the height of the Mode-S antennas to be higher. The results also reveal the benefit of having the antennas provide additional gain in the direction of the runways at the expense of the areas outside the airport perimeter, where there are no targets anyway. The system margin could be further increased by reducing the sensor's MTL with a low-noise high-gain receiver for the radar's front end. If the improved MTL is accompanied by a radar transmitter power greater than 250 W, then a balanced link could be maintained.

Obstructions

At the L-band frequencies of Mode S, buildings that block the line of sight from the radar to the target can obstruct the system's link operation. Thus enough radars must be deployed to guarantee line-of-sight coverage from at least one sensor to every point of interest on the airport surface.

In addition to buildings, large vehicles or aircraft that come between the radar and the target may also degrade link performance. Preliminary qualitative observations indicate that although such degradation does occur, the

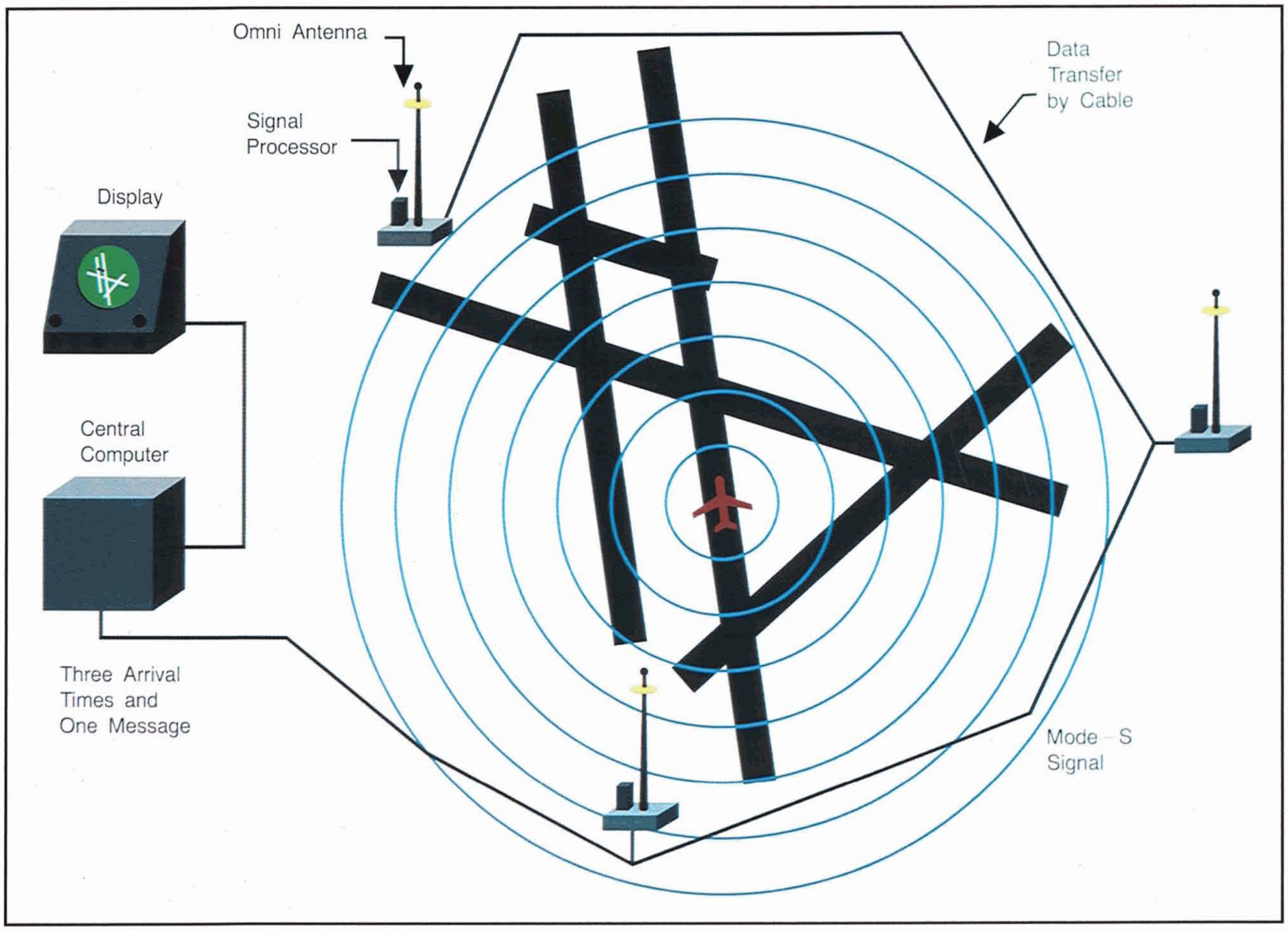

Fig. 4–Candidate system for data link and surveillance.

Table 2. Reply-Link Budget Example

Received Power at Radar		–62 dBm
Transponder Output (P_t) = 250 W	54 dBm	
Cable Loss	–3	
Aircraft Antenna Gain (G_r)	0	
$(4\pi/\lambda)^2$ for $\lambda = 1$ ft	–22	
Range (R) = 5,000 ft	–74	
Radar Antenna Gain (G_t)	0	
$(2\pi/\lambda)^2\ (2^2)$	23	
Aircraft Antenna Height (h_r) = 5 ft	14	
Radar Antenna Height (h_t) = 10 ft	20	
Range (R) = 5,000 ft	–74	
Minimum Trigger Level (MTL)		–74 dBm
Margin		12 dB

problem may be tolerable if sufficient link margin is available.

It is important to note that an airport's central region may have the most stringent coverage requirements because aircraft there often travel at high speeds, which increases the potential for collisions. In such a region, there will most likely be more than one radar able to cover a target. The multiple coverage serves to mitigate the temporary obstructions that vehicles cause.

Building Reflections

From Fresnel theory, it can be shown that a building centered on and at least the size of the first Fresnel zone will reflect nearly all of the transmitted energy if the face of the building is sufficiently reflective. In the geometry of Fig. 5, the 1,000-ft-wide building is wider that the 100-ft Fresnel zone, and the structure's height is a substantial fraction of the zone. The building in the figure could create a reflection that appears at the antenna with considerable amplitude and is delayed by many microseconds relative to the direct signal. Such a reflection could easily destroy the link integrity. All other factors being equal, the reflection would be lower than the direct signal because it would have to travel a greater distance. As it turns out, the relationship between path loss and range to the fourth power for radar operation near the ground (Eq. 1) is helpful in this regard, since the relationship penalizes the reflection with respect to the direct

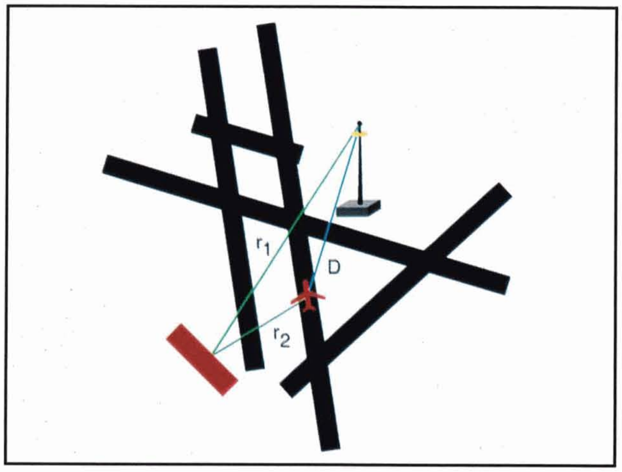

Fig. 5–Reflection geometry.

Fig. 6–Experimental test equipment.

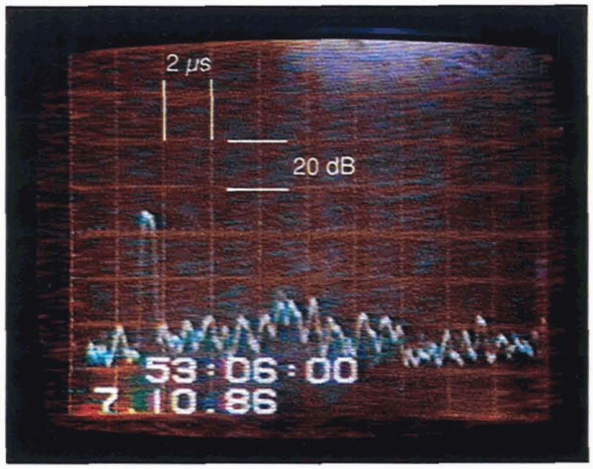

Fig. 8–Directly received pulse with no reflection.

signal much more severely than if the radar, target, and building were located in free space.

Feasibility Study

To test the feasibility of the propagation principles described above, Lincoln Laboratory undertook a brief measurement program at Boston's Logan International Airport. The Laboratory used existing equipment (Fig. 6) that was developed for other FAA programs. A van housed the equipment: an instrumented interrogator/receiver that could transmit and receive ATCRBS and Mode-S signals and record the digitally decoded replies on magnetic tape; and an A-scope display of the receiver-log video channel. A TV camera recorded the A-scope and the interrogator/receiver was connected to a monopole secured on a 1-ft-diameter ground plane mounted 27 ft above the ground. Figure 7 shows the van's location, a site chosen for convenience.

Fig. 7–Van location at Logan International Airport in Boston.

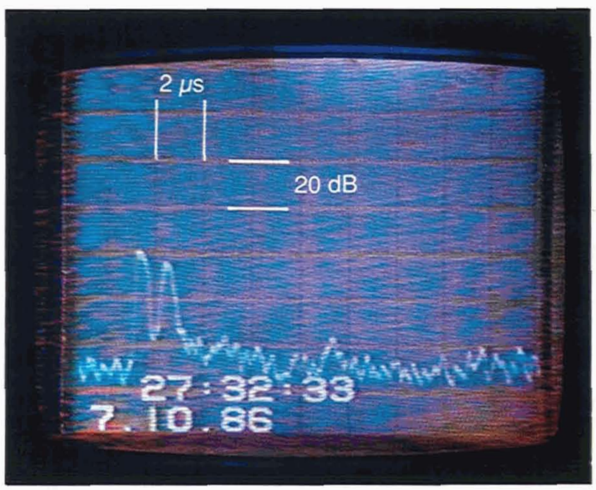

Fig. 9–Directly received pulse with reflection.

During the test, two targets were used. The first target was a yellow pickup truck equipped with a Mode-S transponder (with ATCRBS capability as well) connected to a monopole mounted on a 4-ft-diameter ground plane about 15 ft high. The other target was a Bonanza aircraft equipped with a Mode-S transponder that operated using antennas mounted on the top and bottom of the fuselage. The pickup truck drove around the airport perimeter road and taxiways, and the Bonanza traveled around the taxiways and up and down the runways. A TV camera mounted atop the white van followed the vehicles and indicated obstructing buildings and other factors that could have caused any degradation in link performance.

Reflection Measurements

From the radar measurements of the pickup truck, we can make several observations about the reflection environment. While the truck drove around the perimeter road and taxiways, the van interrogated the truck in ATCRBS mode. The A-scope was set up to display the first pulse in the ATCRBS reply, and any additional energy in the next 20 μs. Figure 8 is a photograph of the TV image of the A-scope when the pickup truck was far from any buildings. Note that the ATCRBS reply pulse is very clean and there are no delayed replicas within about 30 dB of the reply. In contrast, Fig. 9 shows the A-scope when the pickup truck was near a building. Again, the first (or directly received) pulse is clean. This time, however, a replica pulse that is within a few decibels in power from the direct pulse appears one to two microseconds later.

Although the A-scope plots are not enough information to locate the source of the reflection, the data are sufficient to draw an ellipse on which the source of the reflection must lie and to which the face of the reflecting source must be approximately tangent. One focus of the ellipse is the white van with the interrogator/receiver and the other focus is the pickup truck carrying the transponder. The locations of both vehicles are known. The string length with which the ellipse is drawn is the sum of the distance between the foci and the signal delay time (multiplied by the speed of light). This procedure, as applied in Fig. 10, presents very strong evidence that the source of the reflection is the terminal building shown in Fig. 10(a). Note that if the van's interrogator/receiver had the direction-finding capability to determine the angle θ at which the reflection was received, then the location of the reflection source could have been determined by the intersection of the ellipse with a line drawn radially at an angle θ from the van.

Figure 11 shows a more complicated but not unusual case in which the direct pulse is followed by three replicas. After the appropriate ellipses are drawn on an aerial photograph of the airport and interpreted in combination with the TV images recorded by the camera on the van's top, the ellipses strongly indicate that one of the reflections is due to the Federal Express tractor trailers and another to the Eastern Airlines terminal. We could not definitely establish the source of the third replica pulse. Knowledge of θ could have helped our efforts to locate the pulse's source.

Note that computer analysis could have produced a reflection map of the airport if both vehicles had been equipped with direction-finding capabilities as well as navigation systems and sampling circuits that could digitize and record the A-scope on magnetic tape. With such a map, we could have predicted the number and locations of Mode-S radars that were required for a given level of data-link-communications reliability. By knowing the number and locations of the radars, we could have then determined the effectiveness of the trilateration surveillance by calculating the area of the airport for which triple coverage existed under that given network of radars.

Communications-Reliability Measurements

To measure the communications reliability of Mode S on the airport surface, we used the Mode-S interrogate/reply link for both the pickup truck on the perimeter road and the Bonanza on the taxiways and runways. Figure 12, which was made when the Bonanza was far from any buildings, is an example of a clean

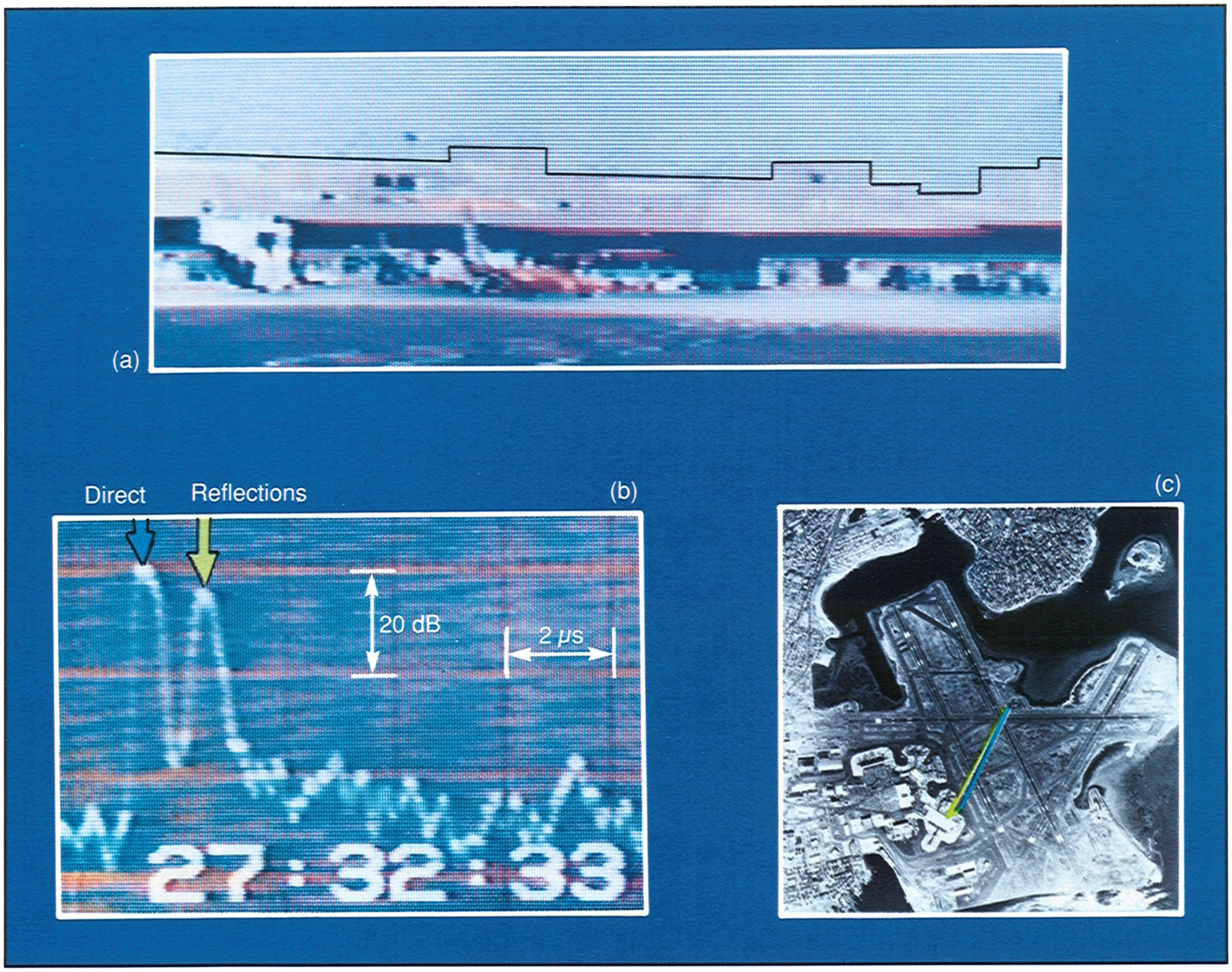

Fig. 10—Example of single reflection. (a) TV view of reflecting building. (b) Received signal showing reflection effect. (c) Aerial view.

Mode-S reply reception. (The overall quality of the figure is poor because the photograph was taken of a TV image.) It is important to note that our method requires the detection and time tagging of just the four-pulse preamble, not the whole waveform. Thus, even if the reply waveform is corrupted, the likelihood that the four-pulse preamble can be detected and time tagged is much greater than the likelihood of correctly decoding all 56 of the PPM-encoded data bits. The disparity in likelihood is even more pronounced if the reply processing circuitry has information regarding the time at which to expect the reply.

Figure 13 shows the results of our tests. In the figure, vehicular paths of travel are indicated by white lines. The Mode-S link performance was greater than 90% everywhere along the paths except for locations marked by blue or black dots. (Performance is defined as the probability of achieving a successful interrogation and reply on a single attempt.) Blue indicates performance between 50% and 90%; black indicates performance below 50%. Most of the degradation in performance was from building reflections. Buildings that caused reflections are shown in red; for each of those buildings, the particular face that caused the difficulty is highlighted in yellow. Using a ruler, a protractor, and Snell's law (which states that the angle of incidence equals the angle of reflection), the curious reader can in many cases identify the building that caused the performance degradation of a particular location.

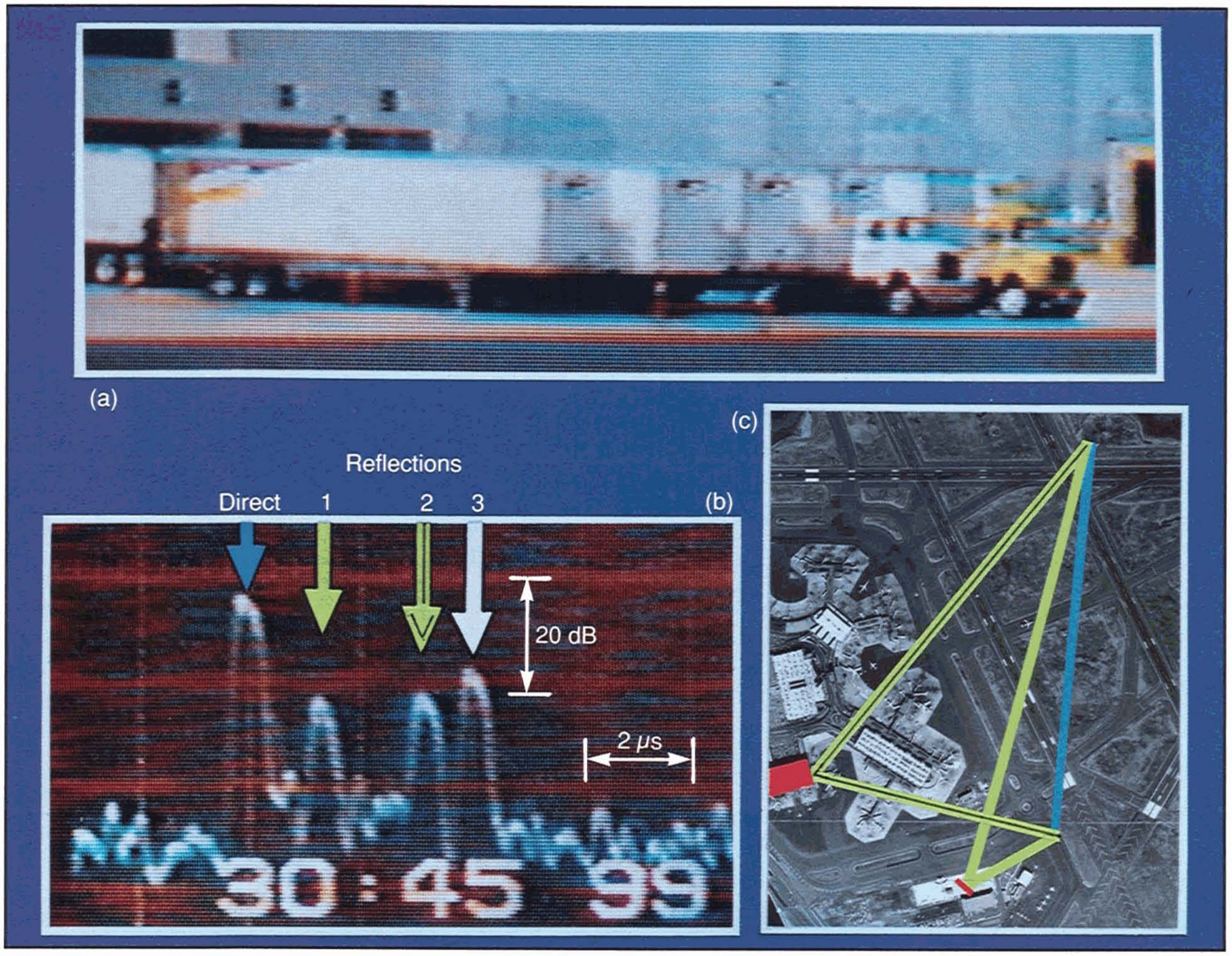

Fig. 11–Example of triple reflection. (a) TV view of reflection source 1. (b) Received signals. (c) Aerial view.

The half dots in Fig. 13 reveal an important point. The Bonanza is a tail-dragger aircraft; i.e., when the plane is taxiing, its fuselage slopes downward from the front to the back. In such a position, the top-mounted antenna can see to the rear better than it can to the front. The half dots represent those times when the Bonanza traveled in a leftward direction down the taxiway and then made a U-turn and returned. In the leftward direction the van was to the rear of the Bonanza and the relative positions of the two vehicles resulted in a direct signal of ample gain. Meanwhile, the reflection, which came from the larger of the two buildings at the far left of the picture, was to the front of the aircraft and consequently was seen with relatively

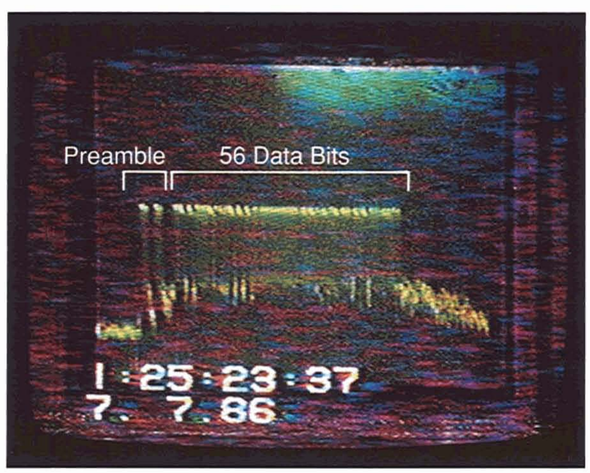

Fig. 12–Mode-S reply.

Fig. 13–Mode-S link reliability.

low gain. On the return trip the situation was reversed. Thus the link performance was considerably worse.

Summary

The Logan Airport measurements were made in two modes: single ATCRBS pulse analysis to determine the reflection environment with signature techniques, elliptical geometry, and aerial photos; and round-trip Mode-S interrogation/reply to determine the link reliability of the system. The measurements showed that most buildings affect performance in a geometrically predictable way. Nevertheless, in a general sense the reliability of Mode-S communications was quite good. We invite the reader to hypothesize whether the particular van site we chose was a good location for an operational Mode-S communications radar and, if so, where the second and third radars should be located to provide coverage in the blue- and black-dot regions. Also, how many radars would be required to eliminate all of the blue and black dots?

Acknowledgments

The author thanks J.V. Gagnon, R.K. Halvorsen, and G.S. Rezendes for their skillful construction, setup, and operation of the experimental test facilities at Boston's Logan International Airport; the Laboratory Photo Lab, which provided much-appreciated and invaluable technical support during and after the measurements; and W.H. Harman for his collaboration in the theoretical design of the experiments and the analysis of the results. The author also thanks the Lincoln Laboratory Flight Facility and its pilots under the direction of E.R. Becotte.

This work was sponsored by the Federal Aviation Administration.

References

1. V.A. Orlando, "The Mode S Beacon Radar System," *Lincoln Laboratory Journal*, **2**, p. 345 (1989).

Appendix: Position Determination

This appendix contains a method for determining the position of a target from three radar ground stations. The method requires that the locations of the target and all three radars be in nearly the same plane.

A Simple Case

In Fig. A, one of the three radars lies at the origin of the x- and y-coordinate system of the plane of interest.

Let

(x, y) = location of target,
(x_1, y_1) = location of radar 1,
(x_2, y_2) = location of radar 2,
$(x_3, y_3) = (0, 0)$ = location of radar 3,
d_1 = distance between radar 1 and the origin,
d_2 = distance between radar 2 and the origin,
r_1 = distance between radar 1 and the target,
r_2 = distance between radar 2 and the target,
r_3 = distance between radar 3 and the target,
t_1 = time that radar 1 receives the signal,
t_2 = time that radar 2 receives the signal,
t_3 = time that radar 3 receives the signal,
c = speed of light,
$\delta_1 = (t_3 - t_1)c$, and
$\delta_2 = (t_3 - t_2)c$.

Then

$$\delta_1 = r_3 - r_1, \text{ so that } r_1 = r_3 - \delta_1.$$

Squaring both sides of the above equation yields

$$r_1^2 = r_3^2 - 2r_3\delta_1 + \delta_1^2.$$

From the geometry of Fig. A

$$r_1^2 = (x - x_1)^2 + (y - y_1),$$
$$r_3^2 = x^2 + y^2, \text{ and}$$
$$d_1^2 = x_1^2 + y_1^2.$$

Appropriate manipulation yields

$$r_3 = \frac{x_1}{\delta_1}x + \frac{y_1}{\delta_1}y + \frac{\delta_1^2 - d_1^2}{2\delta_1}. \qquad (1)$$

A similar equation exists for the signal that is received at radar 2:

$$r_3 = \frac{x_2}{\delta_2}x + \frac{y_2}{\delta_2}y + \frac{\delta_2^2 - d_2^2}{2\delta_2}. \qquad (2)$$

Setting Eq. 1 and Eq. 2 equal to each other yields

$$\left(\frac{x_1}{\delta_1} - \frac{x_2}{\delta_2}\right)x + \left(\frac{y_1}{\delta_1} - \frac{y_2}{\delta_2}\right)y = \frac{\delta_2^2 - d_2^2}{2\delta_2} - \frac{\delta_1^2 - d_1^2}{2\delta_1}.$$

The preceding equation defines a straight line in the plane that contains the three stations and the target of interest for the simplified case in which the third station lies at $x_3 = y_3 = 0$. The location of the target can be solved with the quadratic formula and a hyperbola whose foci are located at two of the three stations.

The General Case

For the general case in which none of the stations are at the origin, let

(x, y) = location of target,
(x_i, y_i) = location of radar i,
(x_j, y_j) = location of radar j,
(x_k, y_k) = location of radar k,
d_i = distance between radar i and the origin,
d_j = distance between radar j and the origin,
d_k = distance between radar k and the origin,
t_i = signal arrival time at radar i,
t_j = signal arrival time at radar j,
t_k = signal arrival time at radar k,
c = speed of light,
$t_{i,k} = (t_i - t_k)c$,
$t_{k,j} = (t_k - t_j)c$, and
$t_{j,i} = (t_j - t_i)c$.

Then the differential arrival times define a straight line of the form $y = mx + b$, in which m (the line's slope) and b (the line's y-intercept value) are respectively given by

$$m = -\frac{x_i t_{k,j} + x_j t_{i,k} + x_k t_{j,i}}{y_i t_{k,j} + y_j t_{i,k} + y_k t_{j,i}}, \text{ and}$$

$$b = \frac{d_i^2 t_{k,j} + d_j^2 t_{i,k} + d_k^2 t_{j,i} + t_{k,j} t_{i,k} t_{j,i}}{2\left(y_i t_{k,j} + y_j t_{i,k} + y_k t_{j,i}\right)}.$$

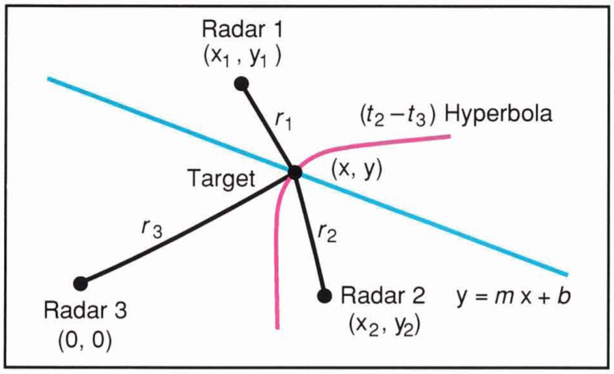

Fig. A—Trilateration geometry.

The target's location can be solved from the intersection of the above straight line with any one of the three hyperbolas whose foci are the radar stations and whose difference-of-distance values are based on the differences of the signal arrival times.

A straight line can intersect a hyperbola in one of two ways: both legs one time each, or one leg twice. Empirically, we have found that some target positions have arrival-time differences that result in the straight line intersecting each of the three hyperbolas in the once-on-each-leg fashion. In other words, no matter which of the three hyperbolas is chosen for intersection with the straight line, the roots of the quadratic will result in two candidate target locations: one on the positive leg, and one on the negative leg. Each hyperbola produces the same two locations. Such target positions enjoy the advantage that only one of the candidate locations can be correct—the one that produces those DTOAs which have the same signs as the measured DTOAs.

We have also found empirically that some target locations result in the straight line intersecting the hyperbolas in the twice-on-one-leg fashion. Such target locations suffer the disadvantage that both of the candidate locations produce DTOAs with the same signs as the measured DTOAs. Consequently, neither can be identified as being the correct target location unless additional information is available.

Although two-way range measurements are not accurate enough, because of transponder turn-around-time uncertainties, to use for target-position measurements, they can in many cases be used to determine which of the trilateration solutions is the true one. In a similar fashion, although large, highly accurate antennas that measure bearing are usually unaffordable, simpler, less accurate ones can be used to select the true trilateration solution.

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