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A THEORY FOR OPTIMAL MTI DIGITAL SIGNAL PROCESSING
SUPPLEMENT I

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A Theory for Optimal MTI Digital Signal Processing

Supplement I

ABSTRACT

In the report, "A Theory for Optimal MTI Digital Signal Processing, Part I: Receiver Synthesis," [1], the problem of eliminating scanning ground clutter from an aircraft surveillance radar was examined from a statistical decision theoretical point of view. An optimum processor was derived which could be approximated by a clutter filter followed by a discrete Fourier transform (DFT). In this report, additional numerical work is documented that compares the performance of the pulse cancellers, pulse cancellers with feedback and the DFT with that of the optimum processor. The issue of coherent vs incoherent integration gain is considered by comparing the filters only on their ability to reject clutter. A clutter rejection improvement factor is defined and used to compare the various filters. It is shown that the pulse cancellers can be quite effective in rejecting clutter provided the input clutter power is not too large and that additional gains are possible using the DFT.

Accepted for the Air Force
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I. INTRODUCTION

In the report, "A Theory for Optimal MTI Digital Signal Processing, Part I: Receiver Synthesis," [1], the problem of eliminating scanning ground clutter from an aircraft surveillance radar was examined from a statistical decision theoretic point of view. In this way, an optimum MTI processor was derived whose performance could be used as a benchmark to compare practical receivers that have been in use for the last two decades. Furthermore, it was of interest to determine whether or not digital processing techniques would be of any use in improving the ability of a radar to reject clutter. It was found that the optimum filter could be interpreted as a clutter filter followed by a bank of doppler filters matched to the two-way antenna scanning modulation. It was suggested that a good approximation to the optimum processor might be a classical clutter filter followed by a discrete Fourier transform (DFT). This would then provide the link between digital signal processing techniques and improved clutter rejection.

It was originally intended that Part I be principally a theoretical document to demonstrate the thought process linking the digital processing of data to MTI clutter rejection and to show the derivation of the tools needed to effect a comparison of the old schemes with the new. In our haste to get the ideas in print, a figure was drawn which compared the performance of the pulse canceller MTI filters with the optimum performance possible. It was intended to show how the signal-to-interference ratio (SIR) performance criterion could be used to evaluate filter performance. Unfortunately, a conclusion was drawn from the curves which has become quite controversial. In fact, the comparison was somewhat unfair because the optimum processor was permitted full use of coherent integration gain, while the pulse cancellers were evaluated allowing for no incoherent averaging. Of course, if the clutter is of such a level that the canceller leaves little residual clutter, then there will be little loss in using incoherent, rather than coherent integration since the number of pulses available for integration is small. On the other hand, if the clutter saturates the cancellers, such that significant residual clutter is produced, then incoherent integration ought to result in little improvement in the overall performance.

To clarify these issues we have performed more numerical work to compare the performance of the pulse cancellers, pulse cancellers with feedback and the DFT with that of the optimum processor. This is done in Sections II and III. Then, in Section IV, we address the issue of coherent vs incoherent integration gain, by comparing the filters only on their ability to reject clutter. We define a clutter rejection improvement factor and compare the various filters once again. It is shown that the pulse cancellers can be quite effective in rejecting clutter provided the input clutter power is not too large and that additional gains are possible using the DFT.

II. PRELIMINARY DEFINITIONS

In this section, we plan to perform a more detailed comparison of the performance of many of the MTI filters that are found in practice. The criterion on which this comparison is based is the signal-to-interference ratio (SIR) derived in Part I, [1]. For the optimum linear processor it was shown, in Eq. (89), that the SIR was given by

$$\rho_{opt}(\nu_o) = \frac{|\gamma_o|^2}{2N_o T_p} \int_{-1/2T_p}^{1/2T_p} \frac{|F_g(f - \nu_o)|^2}{\left[\frac{\sigma_c^2}{2N_o} \frac{1}{T_E T_p} |F_g(f)|^2 + 1\right]} \, df \quad , \tag{1}$$

where $F_g(f)$ is the Fourier Transform of the two-way antenna pattern and

$\frac{|\gamma_o|^2}{N_o}$ = predetection signal-to-noise ratio (SNR),

$\frac{\sigma_c^2}{N_o}$ = predetection clutter-to-noise ratio (CNR),

ν_o = target Doppler,

T_p = interpulse period,

T_E = effective time on target,

$= \theta_B/\omega_s$,

θ_B = one-way antenna 3 dB beamwidth,

ω_s = rate of antenna scan.

It was shown that the optimum filter could be realized as a clutter filter followed by a Doppler filter bank. For any other linear filter the SIR performance was shown to be given by

$$\rho_{sub}(\nu_o) = \frac{|\gamma_o|^2}{2N_oT_p} \cdot \frac{\left|\int_{-1/2T_p}^{1/2T_p} H(f)F_g(f - \nu_o)\, df\right|^2}{\frac{\sigma_c^2}{2N_o} \cdot \frac{1}{T_pT_E} \cdot \int_{-1/2T_p}^{1/2T_p} |H(f)|^2|F_g(f)|^2\, df + \int_{-1/2T_p}^{1/2T_p} |H(f)|^2\, df} , \tag{2}$$

where H(f) is the transfer function of the filter of interest.

All of the results that follow are based on a Gaussian antenna pattern. In this case, the one-way antenna voltage pattern is

$$G(\theta) = e^{-\left(\frac{\theta}{\Delta\theta}\right)^2} , \tag{3}$$

where $\Delta\theta$ is chosen to make the 3 dB beamwidth θ_B. From this we compute the two-way pattern as

$$g(t) = G(\omega_s t) , \tag{4}$$

and taking its Fourier Transform we obtain

$$F_g(f) = \sqrt{\frac{2}{\pi}} \quad e^{-2\alpha^2 f^2} \quad , \tag{5}$$

where $\alpha = \frac{\pi}{2} \frac{\Delta\theta}{\omega_s}$. The system parameters used in all of the comparisons are those used in the FAA Airport Surveillance Radar. They are:

$$\begin{aligned} T_p &= 1/1200 \text{ sec}, \\ \theta_B &= 1.5 \text{ deg}, \\ \omega_s &= 15 \text{ rpm}. \end{aligned} \tag{6}$$

The SNR parameter is chosen such that in the absence of clutter the SIR of the optimum processor is 0 dB. For the above parameter values this requires that the SNR be -8.75 dB.

In the next section, we will specify several MTI filters of current interest and compare their performance with the optimum as a function of target Doppler and CNR.

III. MTI FILTER SPECIFICATION

In this section, we shall briefly review the MTI filters that will be used in the comparison. Then in Section IV, their performance will be compared in a variety of operating environments.

A. The Optimum Filter

In Part I it was shown that the best detection performance was achieved by the filter having the transfer function

$$H(f) = \frac{F_g^*(f - \nu_o)}{\frac{\sigma_c^2}{T_E} |F_g(f)|^2 + 2N_oT_p} , \tag{7}$$

provided the true target Doppler is ν_o. Using a bank of these filters then gives an upper bound on the SIR that can be achieved by the class of linear processors. This bound is given by (1). In addition to the clutter rejection properties of this filter, the overall performance is enhanced by the target matched filter which provides the maximum coherent integration gain for the target in receiver noise.

B. The Pulse Cancellers

In Part I, it was shown that the denominator in (7) could be interpreted as a clutter filter as it produced a null about DC. Although optimum, this would be hard to realize in practice because it requires precise knowledge of the antenna pattern and the average clutter power. Based on classical theory it seems reasonable to approximate this clutter filter by the pulse canceller filters that have the transfer function

$$H(f) = \left(1 - e^{-j2\pi f T_p}\right)^{n_c} , \tag{8}$$

where $n_c + 1$ is the number of pulses involved in the cancellation. In other words for the simplest two-pulse canceller $n_c = 1$. Since

$$|H(f)| = |\sin \pi f T_p|^{n_c} , \tag{9}$$

the pulse cancellers locate a zero at DC and in addition, as n_c increases, the width of the null increases.

C. Feedback Cancellers

Although the above clutter filters can effectively eliminate clutter, the price paid is a loss in signal detectability because of the overall poor shaping of the velocity response curve. In order to regain some of this loss in detectability, feedback is introduced to shape the overall response curve. It is obvious that the best clutter filter would provide a wide notch about DC to null out the clutter and then a flat response elsewhere. This type of response curve can be achieved using feedback. A common realization is the dual delay-line canceller with feedback. This has the transfer function

$$H(f) = \frac{(z - 1)^2}{z^2 - (\alpha_1 + \alpha_2) z + \alpha_1} , \tag{10}$$

where $z = e^{j2\pi fT_p}$. It is expected that as the response is shaped to give better target detectability the clutter rejection capabilities will degrade because the depth of the notch about DC must move as the bulk of the response moves upward.

D. The DFT Processor

The optimum processor was shown to be a clutter filter in cascade with a Doppler filter bank. In addition to the difficulty is realizing the optimum clutter filters the velocity filters would be very difficult to construct using analog hardware especially if many range gates are to be considered. Using digital hardware, however, the problem becomes tractable since the Doppler filter bank is well approximated by a Discrete Fourier Transform (DFT). If the data is first passed through a standard pulse canceller before the DFT is taken, we should have a fairly good approximation to the optimum filter. In this case, if $r(nT_p)$ represents samples of the incoming data, and $r_c(nT_p)$ the output of the clutter filter, then the N-point DFT of this latter sequence yields the frequency samples

$$\xi(kT_p;m\Delta\nu) = \sum_{n=k-N+1}^{k} r_c(nT_p) g\left[\left(n + \frac{N}{2} - k\right) T_p\right] e^{-j2\pi\frac{nm}{N}} , \tag{11}$$

where $\Delta\nu = 1/NT_p$. This can be expressed as the output of a filter whose impulse is

$$h(nT_p;m\Delta\nu) = w(nT_p)\ g\left[\left(- n + \frac{N}{2}\right) T_p\right] e^{j2\pi\frac{nm}{M}} , \tag{12}$$

where

$$w(nT_p) = \begin{cases} 1 & 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases} \quad . \tag{13}$$

Furthermore, the output of the clutter filter is

$$r_c(nT_p) = \sum_{k=-\infty}^{\infty} r(kT_p)\, h_c[(n - k)\, T_p] \tag{14}$$

where $h_c(kT_p)$ is the sampled-data impulse response of any one of the previously described filters. Then the overall DFT-clutter filter processor has the transfer function

$$H(f;m\Delta\nu) = \frac{1}{T_p} H_c(f)\, F_{wg}(f - m\Delta\nu) \ , \tag{15}$$

where $F_{wg}(f)$ is the Fourier Transform of the waveform $w(t)\, g(- t + \frac{N}{2})$. It is worth noting that the pulse canceler frequency response changes slowly relative to that of $F_{wg}(f - m\Delta\nu)$. Therefore, the detection performance of the processor can be improved with no loss in clutter rejection by normalizing each of the DFT coefficients by $H(m\Delta\nu)$. Therefore, the approximation to the optimum MTI processor is taken to be

$$H(f;m\Delta\nu) = \frac{1}{T_p} \cdot \frac{H_c(f)}{H_c(m\Delta\nu)} \cdot F_{wg}(f - m\Delta\nu) \quad . \tag{16}$$

This expression is used in (2) to generate its SIR performance. In the results to follow we shall take $H_c(f)$ to be the three pulse canceler. (i.e., n_c=2).

IV. COMPARISON OF PERFORMANCE

In the last section, several MTI filters of theoretical and practical interest were proposed. In this section, curves showing their SIR performance vs target Doppler for various CNR's will be discussed for the ASR system parameters. We begin with Figure 1 which shows the optimum, two and three pulse cancellers and the DFT processor for a CNR of 48 dB. The curves show that the DFT-3 pulse canceller is a good approximation to the optimum. It appears that the classical pulse cancellers are performing significantly poorer than the DFT processor. However, part of this performance loss is due to the fact that the DFT implicitly utilizes coherent integration gain since each DFT coefficient represents the output of a perfectly matched filter. Since the pulse cancellers will undoubtedly be followed by some incoherent integration of pulses or at least by an operator at a cathode ray tube, the SIR performance measure is an unfair criterion for comparing the clutter rejection capabilities of the various filters. It is useful in evaluating various DFT processors (i.e., using fewer data samples) as the degradation from the overall optimum SIR performance can then be determined directly. However, to fairly compare the pulse canceller with the DFT processor, we adopt another performance measure, the output peak signal to average clutter ratio (SCR). This is obtained from (1) and (2) by neglecting the effect of filtering the receiver noise. In this case, the optimum performance is given by

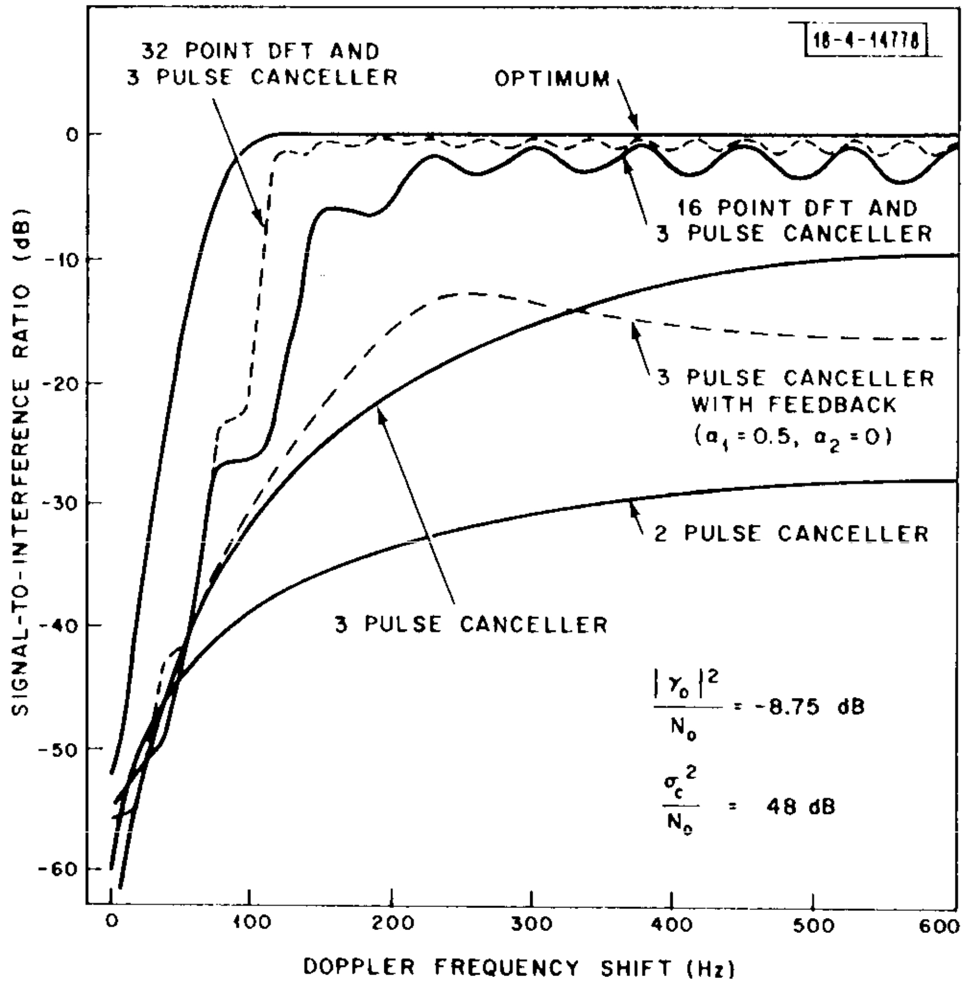

Figure 1. Signal-to-interference ratio for several practical MTI processors.

$$\beta^{o}_{opt}(\nu_o) = T_E \cdot \frac{|\gamma_o|^2}{\sigma_c^2} \cdot \int_{-1/2T_p}^{1/2T_p} \frac{|F_g(f - \nu_o)|^2}{|F_g(f)|^2} \, df \quad , \tag{17}$$

while that of the suboptimal processors is given by

$$\beta^{o}_{sub}(\nu_o) = T_E \cdot \frac{|\gamma_o|^2}{\sigma_c^2} \cdot \frac{\left| \int_{-1/2T_p}^{1/2T_p} H(f) \, F_g(f - \nu_o) \, df \right|^2}{\int_{-1/2T_p}^{1/2T_p} |H(f)|^2 |F_g(f)|^2 \, df} \quad . \tag{18}$$

It was shown in Part I that the average clutter power per sample was given by

$$\overline{|C(nT_p)|^2} = \frac{\sigma_c^2}{T_E} \int_{-1/2T_p}^{1/2T_p} |F_g(f)|^2 \, df \quad . \tag{19}$$

Therefore, the input peak signal-to-clutter ratio is

$$\beta^{i} = \frac{|\gamma_o|^2}{\overline{C(nT_p)}^{\,2}} \quad . \tag{20}$$

Then, we define the improvement factor to be

$$I(\nu_o) = \frac{\beta^{o}(\nu_o)}{\beta^{i}} \quad . \tag{21}$$

For the optimum processor this becomes

$$I_{opt}(\nu_o) = \int_{-1/2T_p}^{1/2T_p} |F_g(f)|^2 \, df \int_{-1/2T_p}^{1/2T_p} \frac{|F_g(f - \nu_o)|^2}{|F_g(f)|^2} \, df \ , \tag{22}$$

while the suboptimal processors result in

$$I_{sub}(\nu_o) = \int_{-1/2T_p}^{1/2T_p} |F_g(f)|^2 \, df \cdot \frac{\left| \int_{-1/2T_p}^{1/2T_p} H(f) \, F_g(f - \nu_o) \, df \right|^2}{\int_{-1/2T_p}^{1/2T_p} |H(f)|^2 |F_g(f)|^2 \, df} \ . \tag{23}$$

The improvement factors were computed for the optimum, DFT and pulse canceller processors and the results are shown in Figure 2.

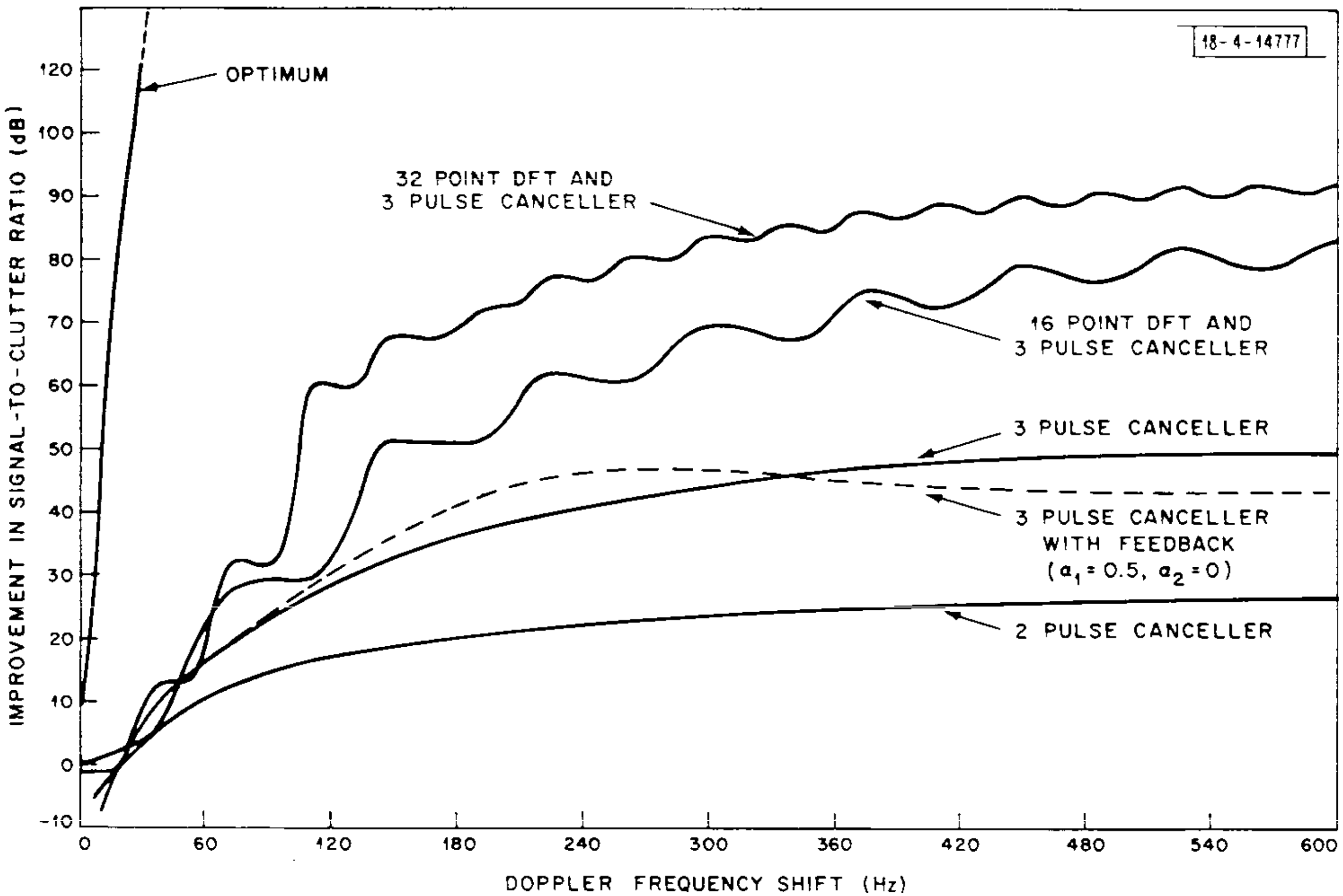

Figure 2. Improvement in clutter reflection by filtering.

V. CONCLUSIONS

In Figure 1, it is clearly demonstrated that shaping the velocity response of the clutter filter can improve the low-frequency performance of the filter at the expense of a greater loss in the high frequency region. Furthermore, the loss in performance is of the order of 15 dB and is due principally to the presence of residual clutter which will not be eliminated by incoherent integration.

Figure 2 shows that the pulse canceller and DFT can be very effective in eliminating scanning ground clutter. This curve shows that much of the improvement shown in the SIR performance curves is due to the ability of the DFT to further reject the residual clutter. By making the data window longer (16 T_p to 32 T_p), the frequency sidelobes of the matched filters are reduced, resulting in less interaction with the residual clutter. This is the principal reason the DFT can lead to significant improvements in the rejection of clutter.

Finally, it can be concluded that if the clutter background is not too severe, then the pulse cancellers can eliminate it effectively. For example, Figure 2 shows that the improvement factor for the three-pulse canceller is more than 30 dB over 75% of the total frequency range. Hence, if the input SCR is at least -15 dB then the output SCR will be +15 dB and the clutter will become a fractional part of the noise background.

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13. ABSTRACT

In the report, "A Theory for Optimal MTI Digital Signal Processing. Part I. Receiver Synthesis," (1), the problem of eliminating scanning ground clutter from an aircraft surveillance radar was examined from a statistical decision theoretical point of view. An optimum processor was derived which could be approximated by a clutter filter followed by a discrete Fourier transform (DFT). In this report, additional numerical work is documented that compares the performance of the pulse cancellers, pulse cancellers with feedback and the DFT with that of the optimum processor. The issue of coherent vs incoherent integration gain is considered by comparing the filters only on their ability to reject clutter. A clutter rejection improvement factor is defined and used to compare the various filters. It is shown that the pulse cancellers can be quite effective in rejecting clutter provided the input clutter power is not too large and that additional gains are possible using the DFT.

14. KEY WORDS

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